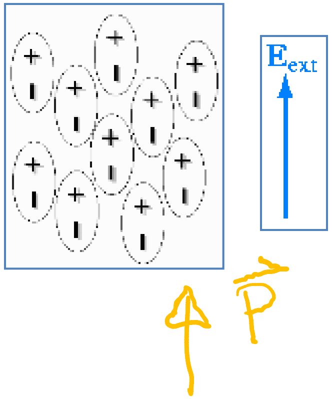
8-1 and 8-2 (*look* at 8-3)

Lecture 33

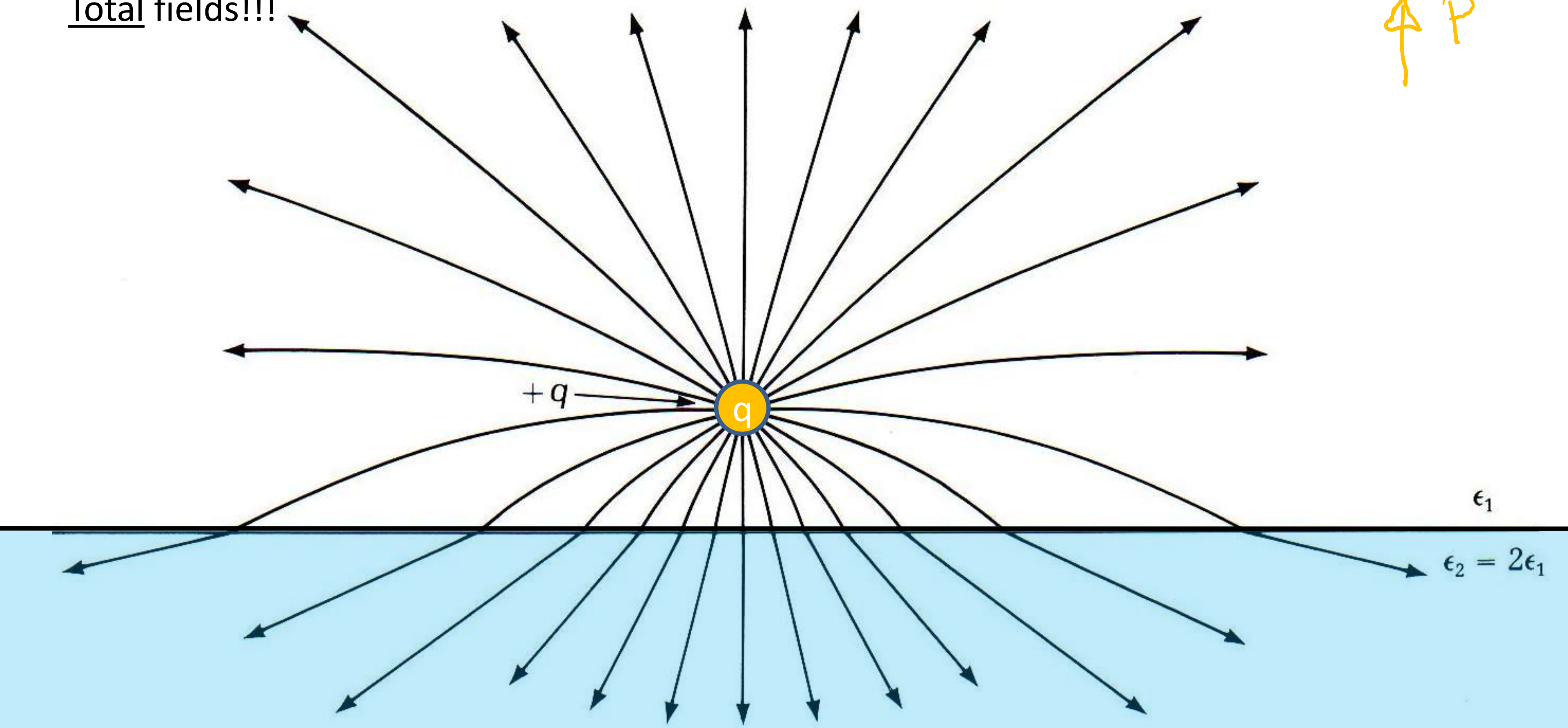
Medium 1 Dielectric	Medium 2 Dielectric

$$E_{1t} = E_{2t}$$
$$D_{1n} - D_{2n} = \rho_s$$
$$H_{1t} = H_{2t}$$
$$B_{1n} = B_{2n}$$

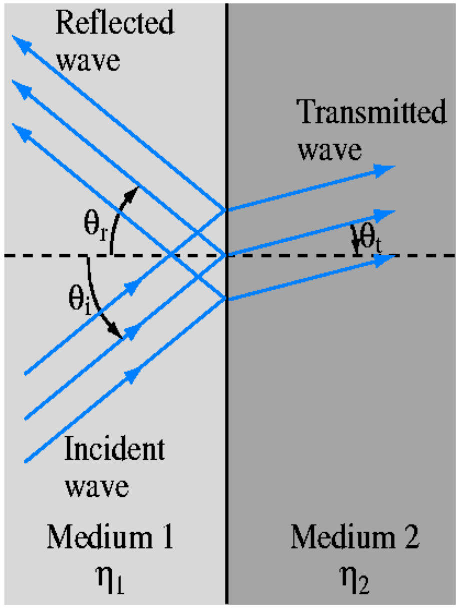
$E_{2n} = \frac{\epsilon_1}{\epsilon_2} E_{1n}$ for $\rho_s = 0$



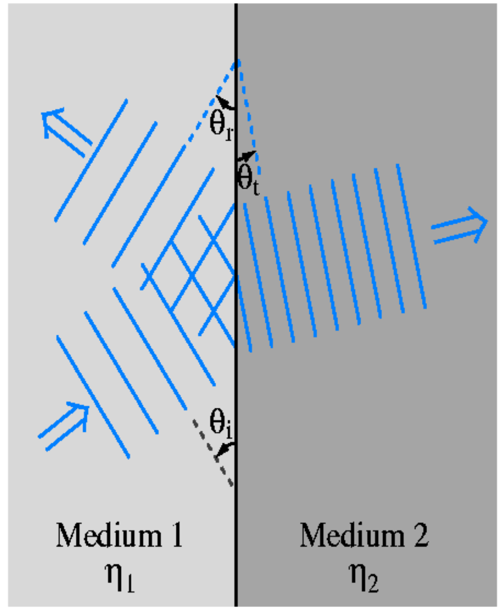
Total fields!!!



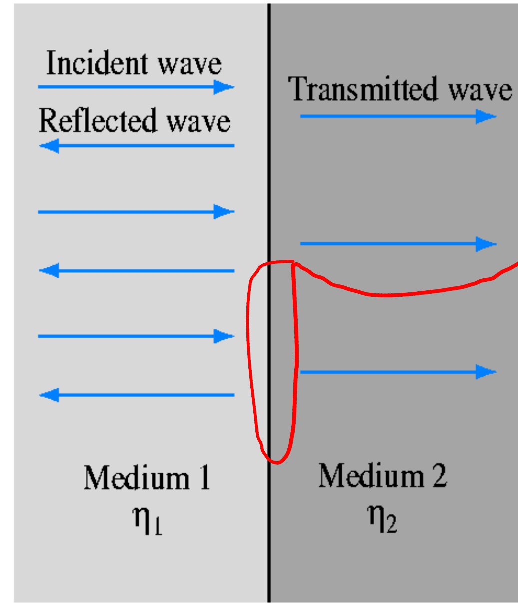
Perfect dielectric



(b) Ray representation of oblique incidence



(c) Wavefront representation of oblique incidence

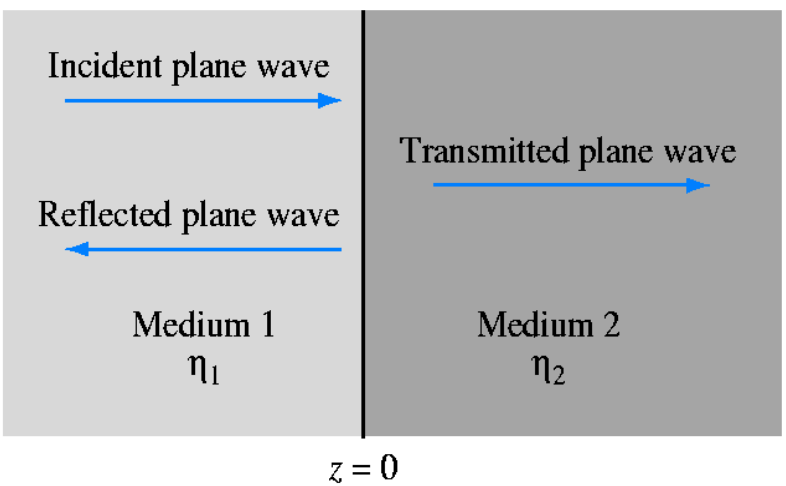


(a) Normal incidence

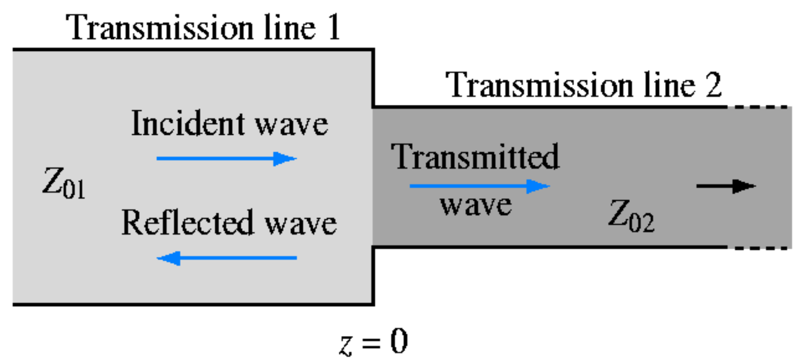
"Optically" SMOOTH *

* "Flat" w/r/t λ

NORMAL Incidence

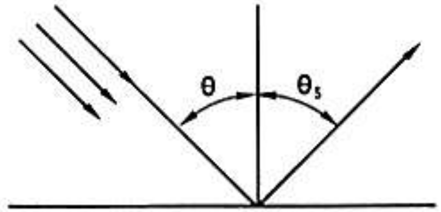


(b) Boundary between different media



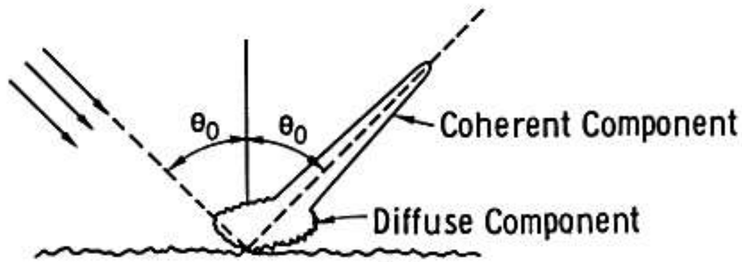
(a) Boundary between transmission lines

What is "smooth" ??



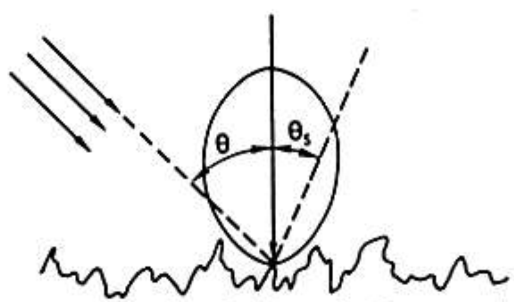
Reflected Power is Entirely Coherent and $\theta_s = \theta$, Scattering Pattern is a Delta Function

(a)



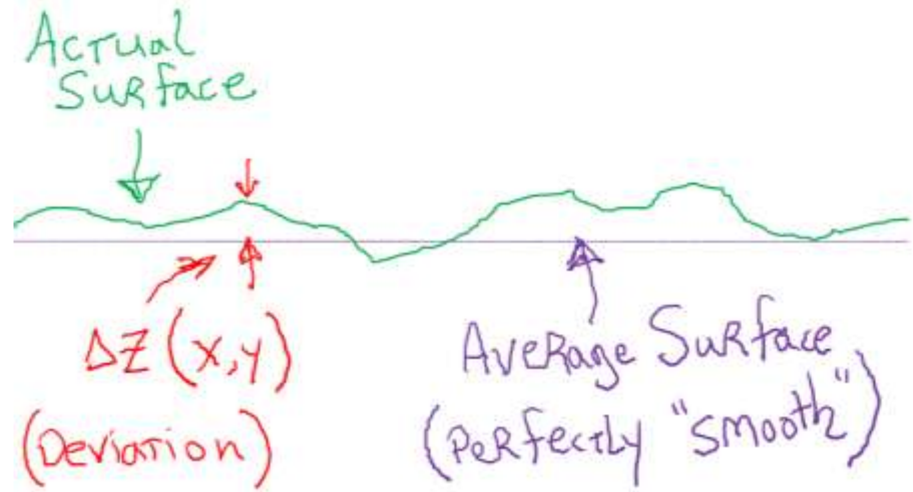
Scattering Pattern Consists of Large Coherent Component and Small Diffuse Component

(b)



Scattering Pattern is Composed Entirely of Diffuse Component. For Lambertian Surface, $\sigma^\circ(\theta, \theta_s) = \sigma_0^\circ \cos\theta \cos\theta_s$

(c)

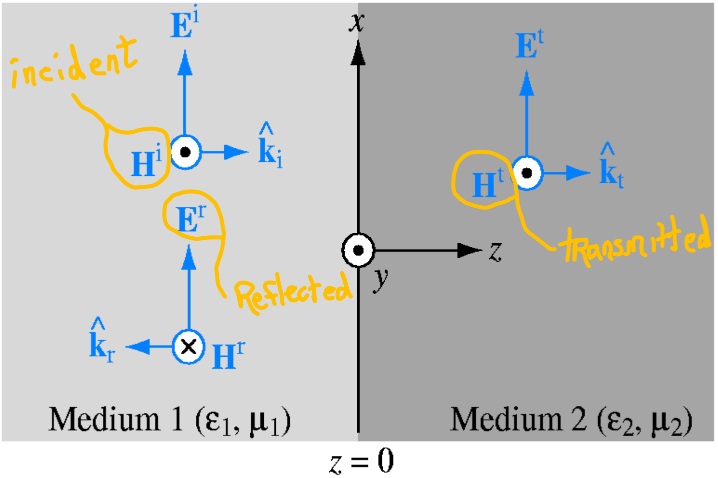


$$RMS = \sqrt{\frac{1}{A}\iint_A (\Delta Z)^2 \, dA}$$

typically scaled to wavelength ...

NORMAL Incidence

Assume no surface charge or current at interface…



$$\hat{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \quad *$$

$$\hat{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = J_S \quad \star$$

$$D_{1n} - D_{2n} = \rho_S$$

$$B_{1n} - B_{2n} = 0$$

$$\eta \equiv \sqrt{\frac{\mu}{\varepsilon}}$$

can depend on freq.

Note: *all* tangential components for normal incidence.

Note: *polarization* doesn't impact results for normal incidence!

Assume $\tilde{E}_i = \hat{x} E_0^i e^{-jk_1 z}$ is known

$*$

$$E_0^i + E_0^r = E_0^t$$

$\star$ and

$$H_0^i + H_0^r = H_0^t$$

$$\Downarrow$$

$$\frac{E_0^i}{\eta_1} - \frac{E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}$$

2 EQTNS.
2 unknowns
(E_0^r, E_0^t)

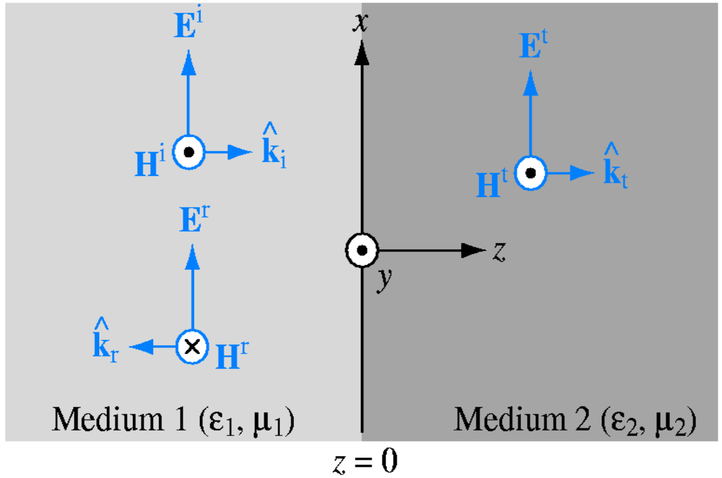
$\Rightarrow$

Γ = Reflection coeff.

$$E_0^r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i$$

$$E_0^t = \frac{2\eta_2}{\eta_2 + \eta_1} E_0^i$$

τ = Trans. coeff.



But what does it *physically* mean?? ☺

Question $\quad Does \quad \Gamma + \tau = 1$??

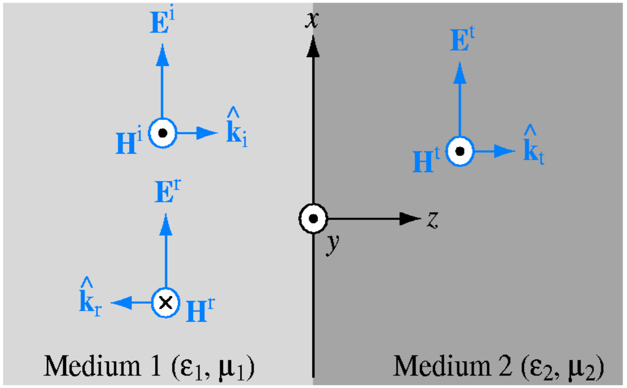
Nope, but...

$$\frac{\left|\Gamma E_o^i\right|^2}{2\eta_1} + \frac{\left|\tau E_o^i\right|^2}{2\eta_2} = \frac{\left|E_o^i\right|^2}{2\eta_1} \Rightarrow$$

For *normal* incidence...

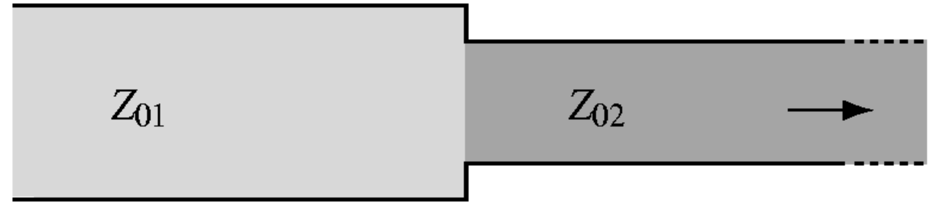
$$\boxed{\left|\Gamma\right|^2 + \frac{\eta_1}{\eta_2}\left|\tau\right|^2 = 1}$$

$$why \left|\Gamma\right|^2, \text{not just } \Gamma^2 \ ?$$



$z = 0$

(a) Boundary between dielectric media

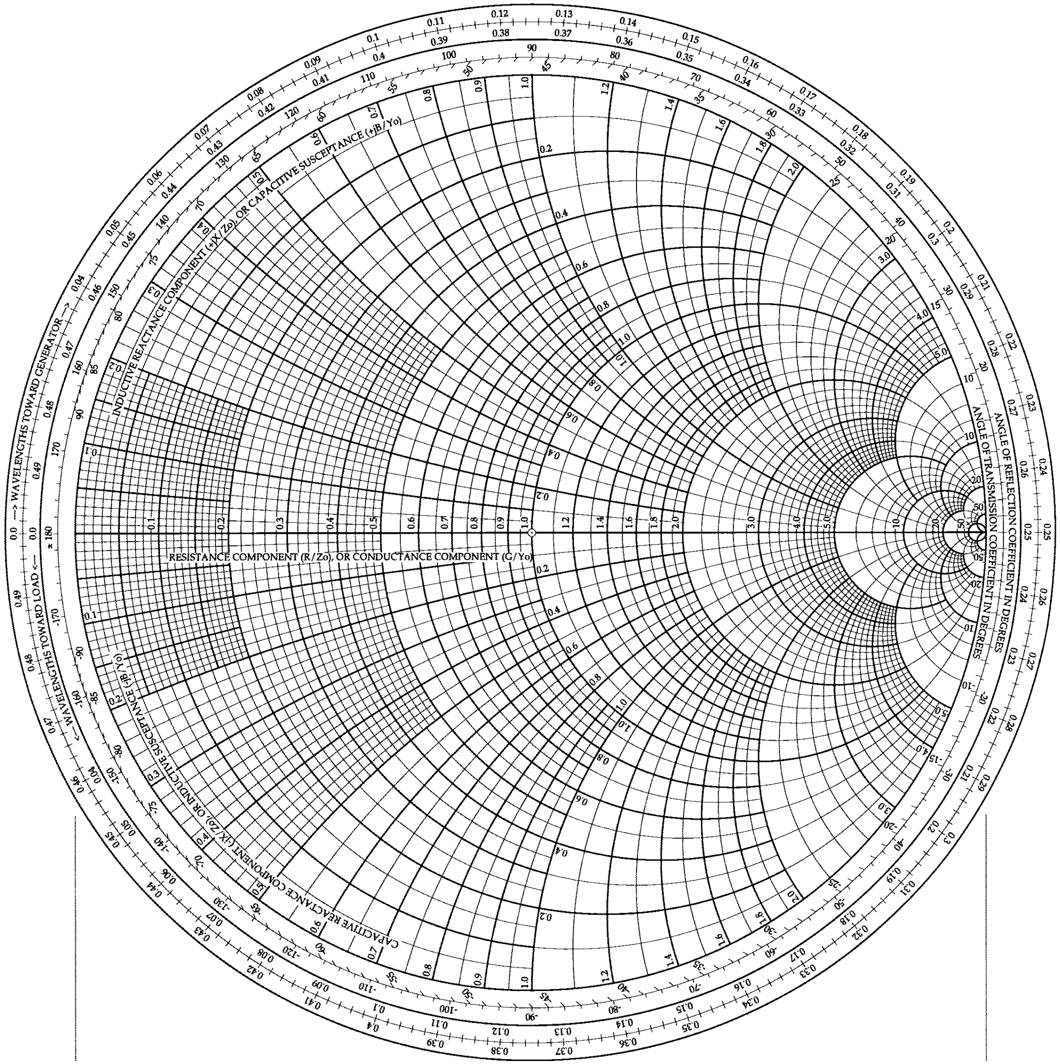


$z = 0$

(b) Transmission-line analogue

Plane Wave [Fig. 8-4(a)]		Transmission Line [Fig. 8-4(b)]	
$\widetilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}} E_0^{\mathrm{i}} (e^{-jk_1 z} + \Gamma e^{jk_1 z})$	(8.11a)	$\widetilde{V}_1(z) = V_0^+ (e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$	(8.11b)
$\widetilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}} \dfrac{E_0^{\mathrm{i}}}{\eta_1} (e^{-jk_1 z} - \Gamma e^{jk_1 z})$	(8.12a)	$\widetilde{I}_1(z) = \dfrac{V_0^+}{Z_{01}} (e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z})$	(8.12b)
$\widetilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}} \tau E_0^{\mathrm{i}} e^{-jk_2 z}$	(8.13a)	$\widetilde{V}_2(z) = \tau V_0^+ e^{-j\beta_2 z}$	(8.13b)
$\widetilde{\mathbf{H}}_2(z) = \hat{\mathbf{y}} \tau \dfrac{E_0^{\mathrm{i}}}{\eta_2} e^{-jk_2 z}$	(8.14a)	$\widetilde{I}_2(z) = \tau \dfrac{V_0^+}{Z_{02}} e^{-j\beta_2 z}$	(8.14b)
$\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$	complex ??	$\Gamma = (Z_{02} - Z_{01})/(Z_{02} + Z_{01})$	
$\tau = 1 + \Gamma$		$\tau = 1 + \Gamma$	
$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$, $\quad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$		$\beta_1 = \omega\sqrt{\mu_1 \varepsilon_1}$, $\quad \beta_2 = \omega\sqrt{\mu_2 \varepsilon_2}$	
$\eta_1 = \sqrt{\mu_1 / \varepsilon_1}$, $\quad \eta_2 = \sqrt{\mu_2 / \varepsilon_2}$		Z_{01} and Z_{02} depend on transmission-line parameters	

Notation varies from text to text!



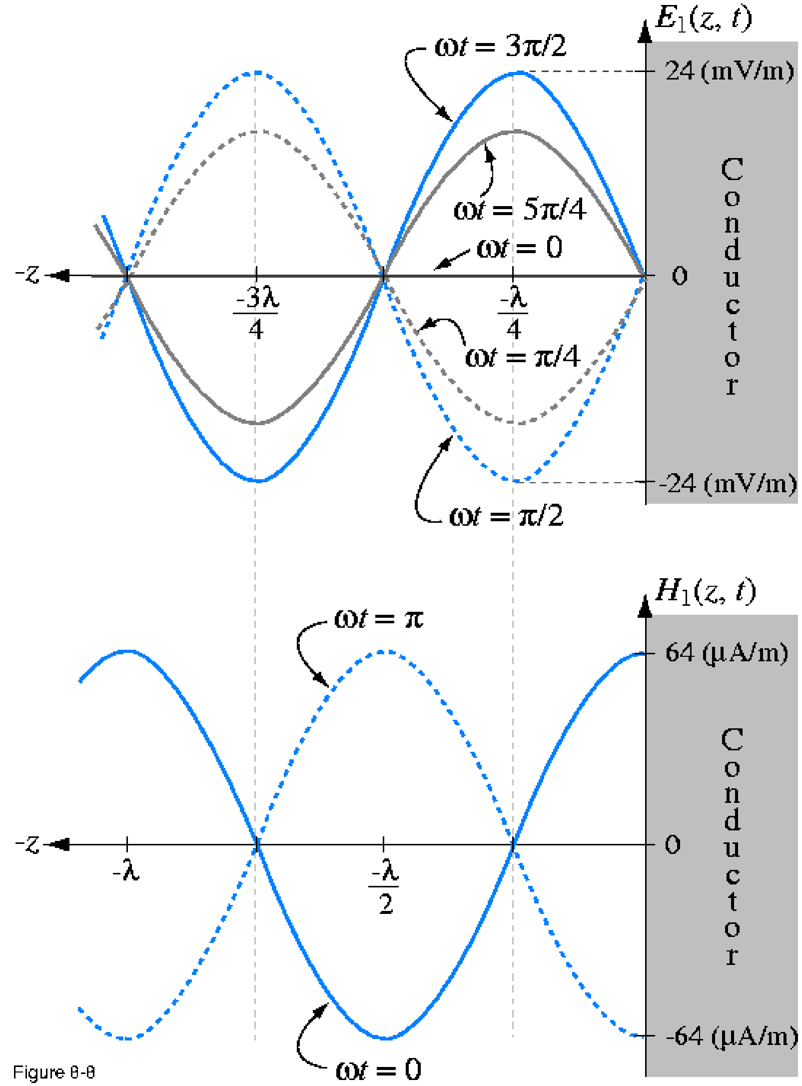


Figure 8-8

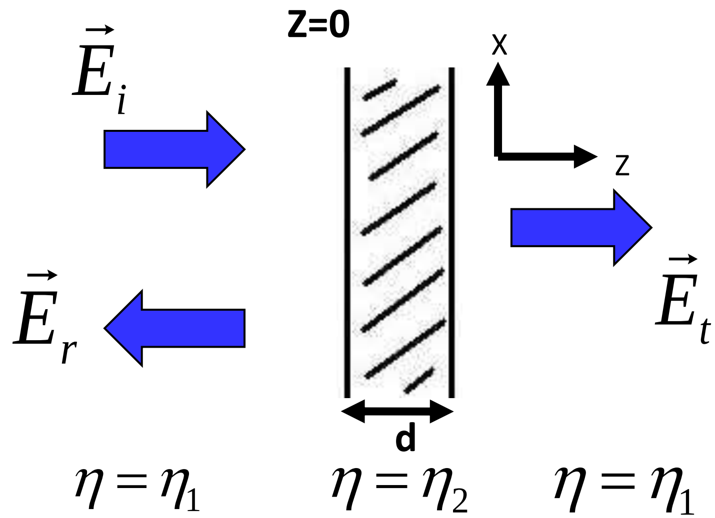
$\epsilon_c = \epsilon_R \epsilon_0 - j\frac{\sigma}{\omega}$

What's this Mean? ☺ → $\Gamma = -1$ ⇐ Perfect Conductor ⇒ $\sigma \to \infty$ ⇒ $\eta_c \to 0\,\Omega$

$\epsilon_c \to -\infty$

Layered Media

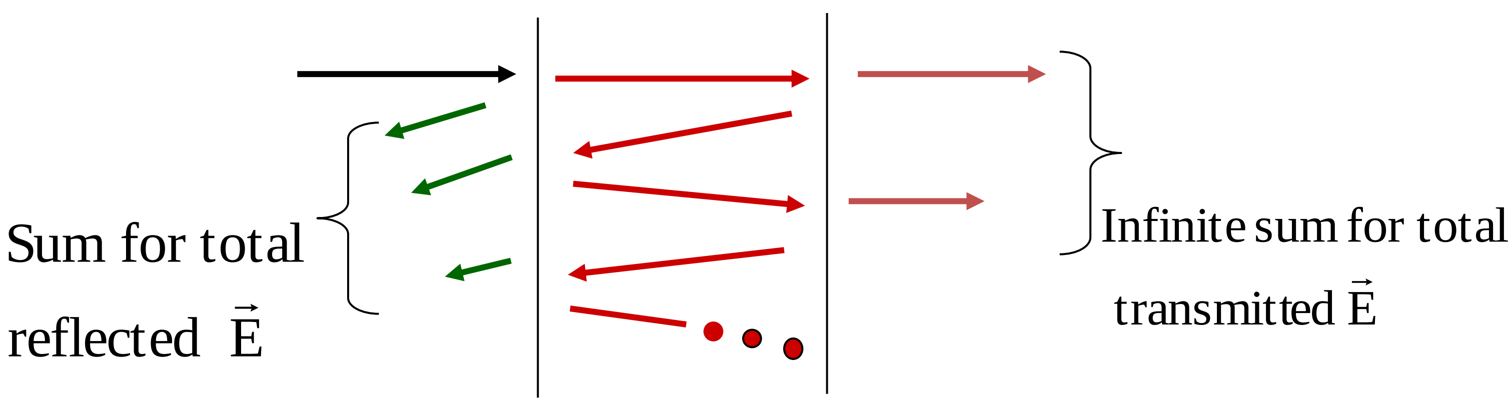
Example: Single Layer!



with:

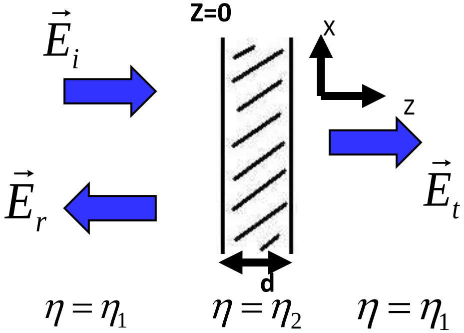
$$\vec{E}_i = \begin{cases} \hat{x}E_o e^{-jkz} & \text{phasor} \\ \hat{x}E_o \cos(\omega t - kz) & \text{time-domain} \end{cases}$$

$$\text{If } \vec{E}_i = \hat{x}E_o \text{ at } z = 0, \vec{E} = \hat{x}E_o e^{-jkd} \text{ at the right side } (z = d)\ldots$$



Superposition!!! *With constructive and/or destructive interference!*

Single Layer (continued)



$$\tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}, \tau_{21} = \frac{2\eta_1}{\eta_1 + \eta_2}$$

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}, \Gamma_{21} = -\Gamma_{12} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$\vec{E}_r = \hat{x}E_i e^{+jkz}\left[\Gamma_{12} + \tau_{12}e^{-jkd}\Gamma_{21}e^{-jkd}\tau_{21} + \tau_{12}(e^{-j2kd})^2\Gamma_{21}{}^2\tau_{21} + \ldots\right.$$
$$\left.\ldots + \tau_{12}\tau_{21}(\Gamma_{21}e^{-j2kd})^2 + \ldots\right]$$

$$= \hat{x}\tau_{12}\tau_{21}E_o e^{+jkz}\left[\frac{\Gamma_{12}}{\tau_{12}\tau_{21}} + \sum_{n=1}^{\infty}\left(\Gamma_{21}e^{-j2kd}\right)^n\right]$$

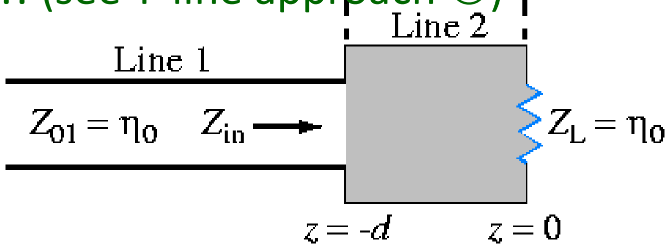
$$= \hat{x}\Gamma_{12}E_o\left[\frac{1 - e^{j2kd}}{1 - \Gamma_{12}{}^2 e^{j2kd}}\right]\mathrm{e}^{+\mathrm{j}kz}$$

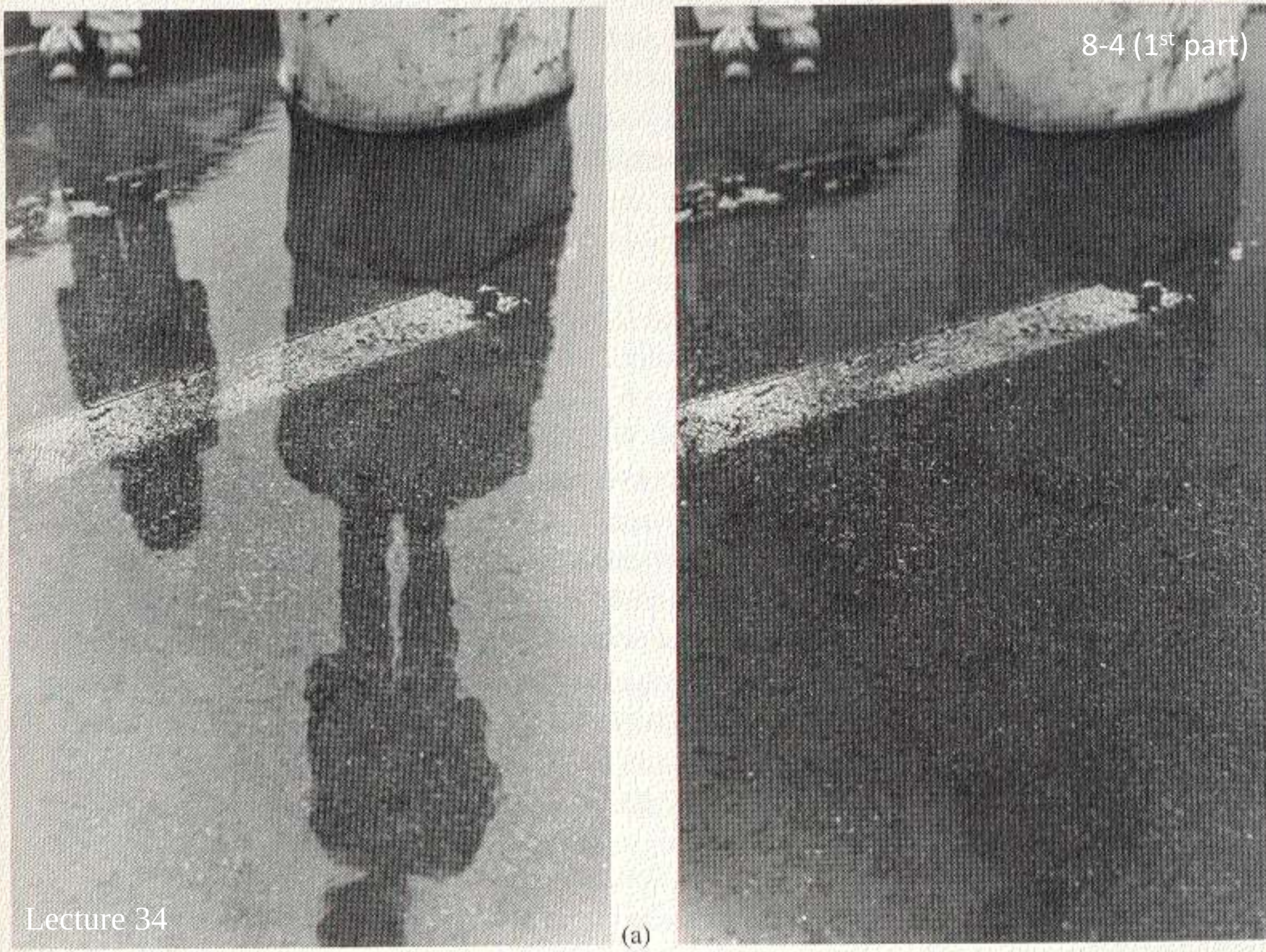
w/ k defined **in** the slab

Note: There is a more elegant way!! (see T-line apprpach ☺)

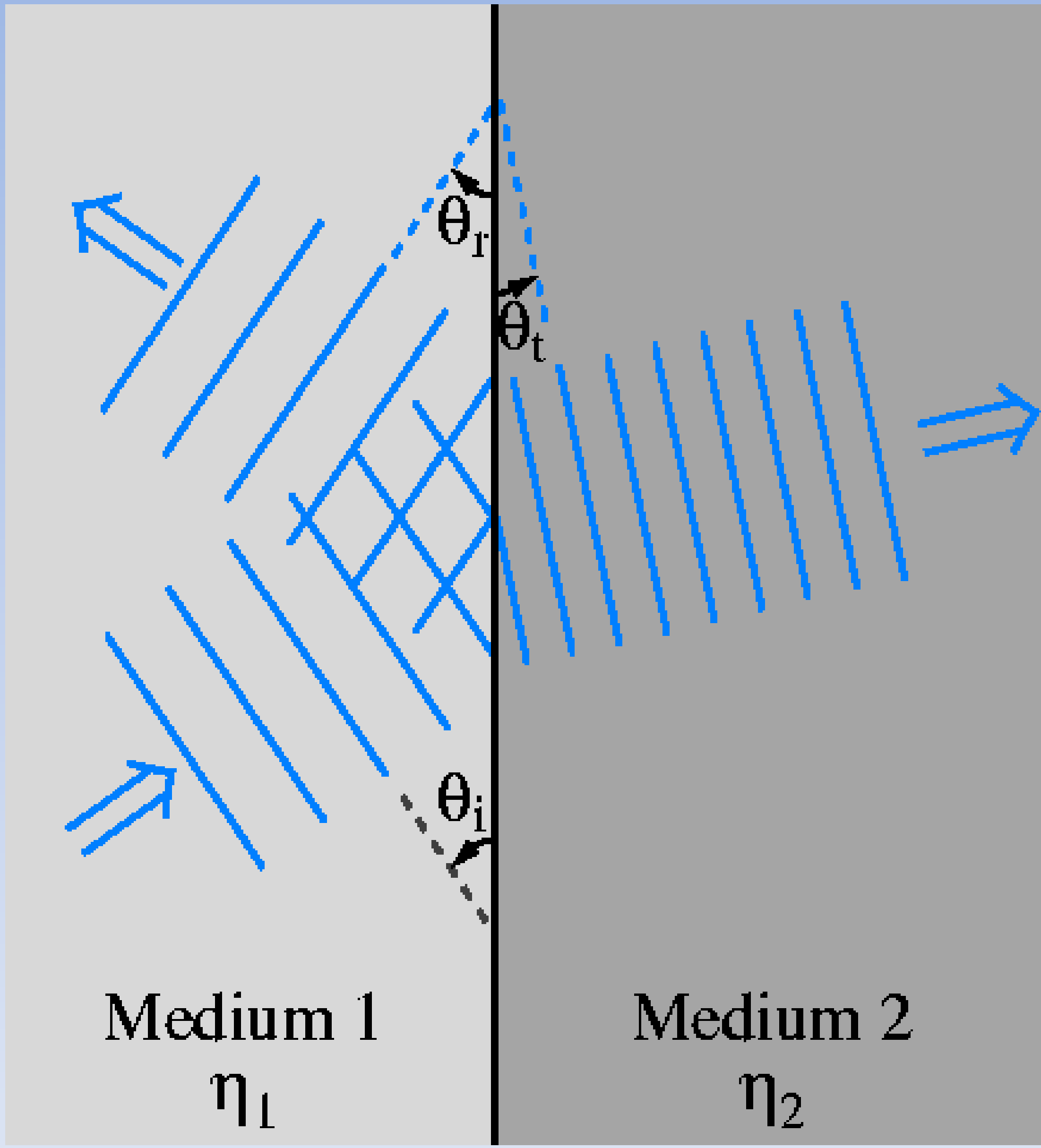
Question:

Can the reflection from a slab ever be zero?





(a) (b)



$$\hat{n}\times(\mathbf{E}_1-\mathbf{E}_2)=0$$

$$\hat{n}\times(\mathbf{H}_1-\mathbf{H}_2)=J_S$$

$$D_{1n}-D_{2n}=\rho_S$$

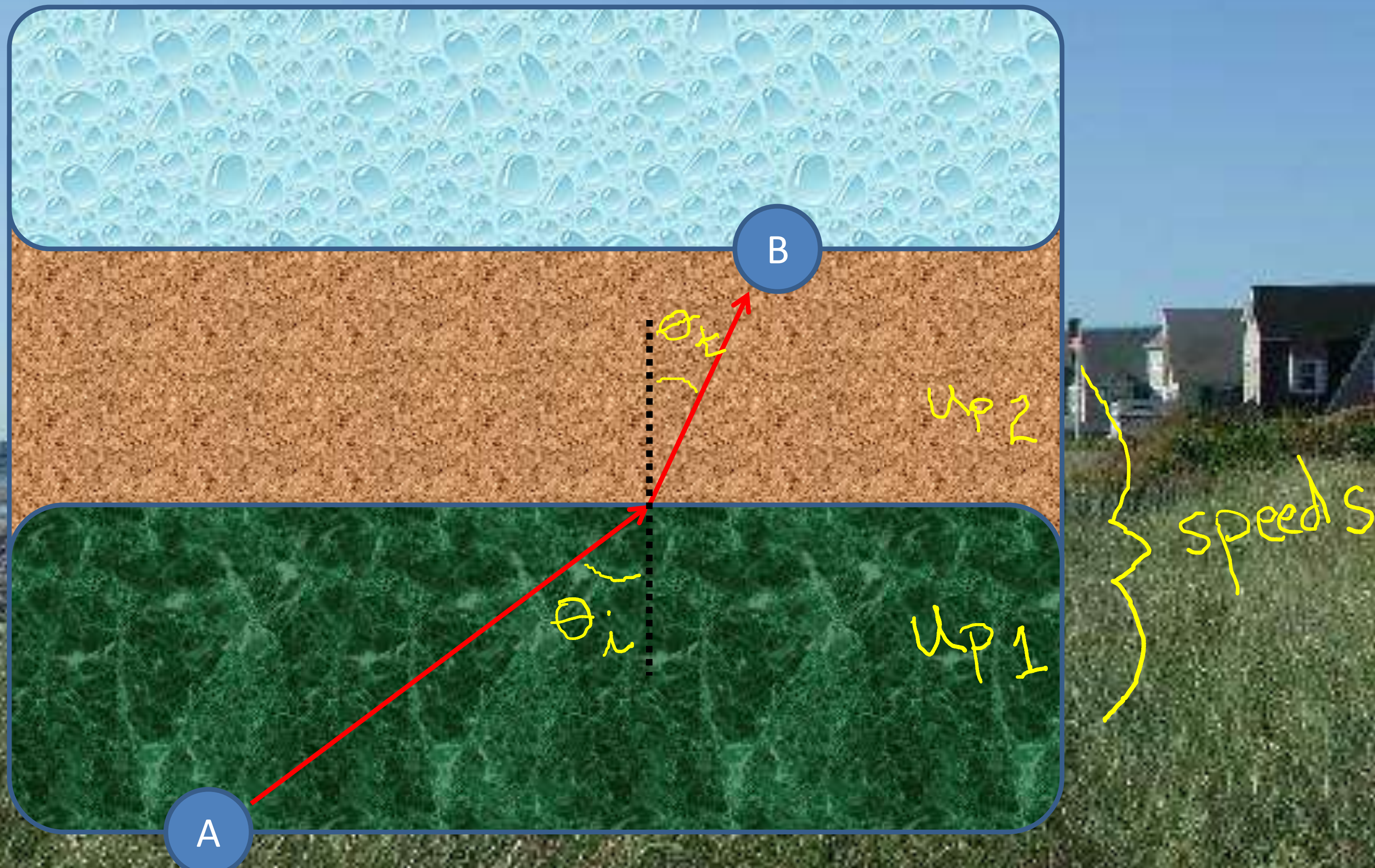
$$B_{1n}-B_{2n}=0$$

$$\theta_r=\theta_i$$

$$\eta_2\sin\theta_i=\eta_1\sin\theta_t$$

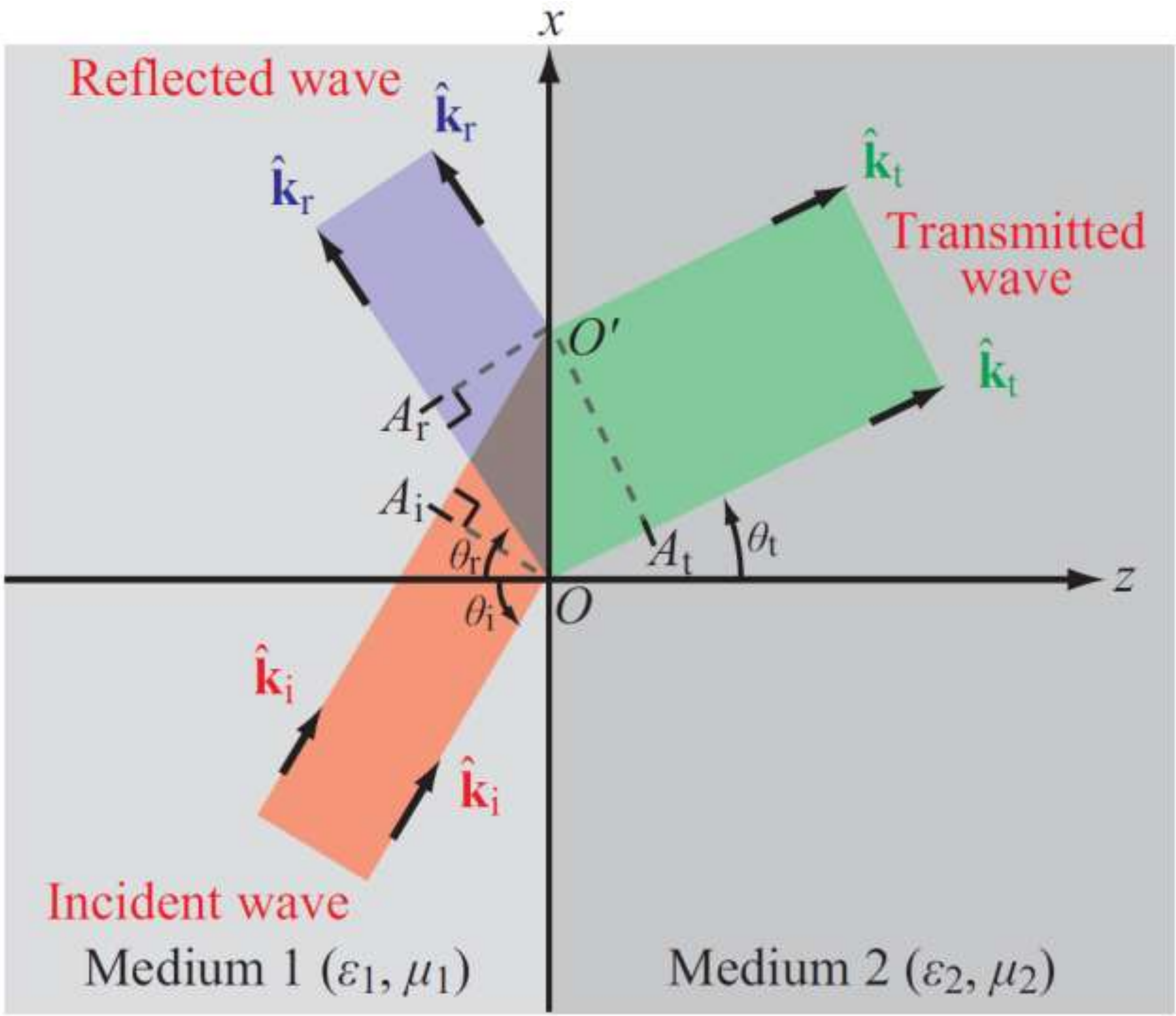
$$\frac{\sin\theta_i}{v_{p1}} = \frac{\sin\theta_t}{v_{p2}}$$

Least Time



http://www.haverford.edu/physics/songs/snell.htm

Snell's Laws



x
Reflected wave
$\hat{\mathbf{k}}_r$
$\hat{\mathbf{k}}_r$
$\hat{\mathbf{k}}_t$
Transmitted wave
$\hat{\mathbf{k}}_t$
O'
A_r
A_i
θ_r
θ_i
A_t
θ_t
z
O
$\hat{\mathbf{k}}_i$
$\hat{\mathbf{k}}_i$
Incident wave
Medium 1 (ε_1, μ_1)
Medium 2 (ε_2, μ_2)

Example 8-4: Light Beam Passing through a Slab

A dielectric slab with index of refraction n_2 is surrounded by a medium with index of refraction n_1, as shown in Fig. 8-11. If $\theta_i < \theta_c$, show that the emerging beam is parallel to the incident beam.

Solution: At the slab's upper surface, Snell's law gives

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

and, similarly, at the slab's lower surface,

$$\sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{n_2}{n_1}\sin\theta_2.$$

Substituting Eq. (8.33) into Eq. (8.34) gives

$$\sin\theta_3 = \left(\frac{n_2}{n_1}\right)\left(\frac{n_1}{n_2}\right)\sin\theta_1 = \sin\theta_1.$$

Hence, $\theta_3 = \theta_1$. The slab displaces the beam's position, but the beam's direction remains unchanged.

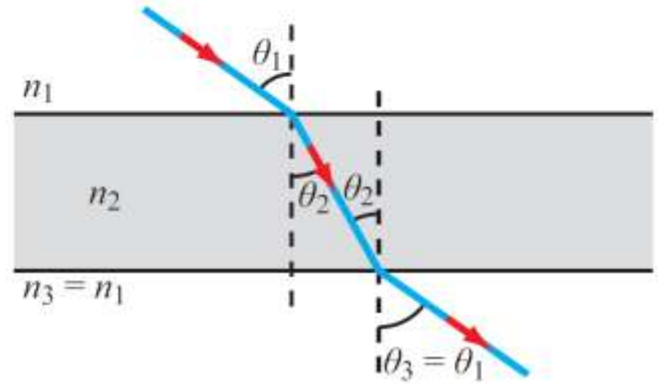
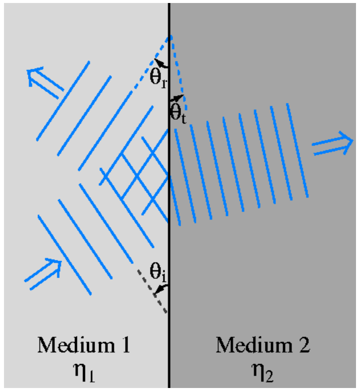
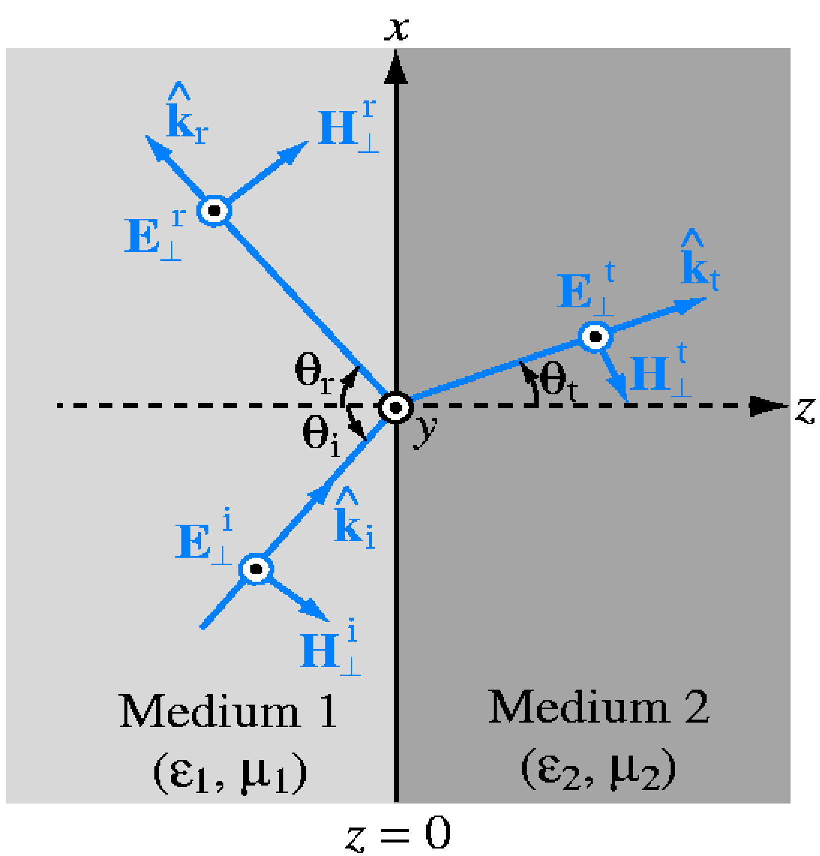


Figure 8-11: The exit angle θ_3 is equal to the incidence angle θ_1 if the dielectric slab has parallel boundaries and is surrounded by media with the same index of refraction on both sides (Example 8-4).



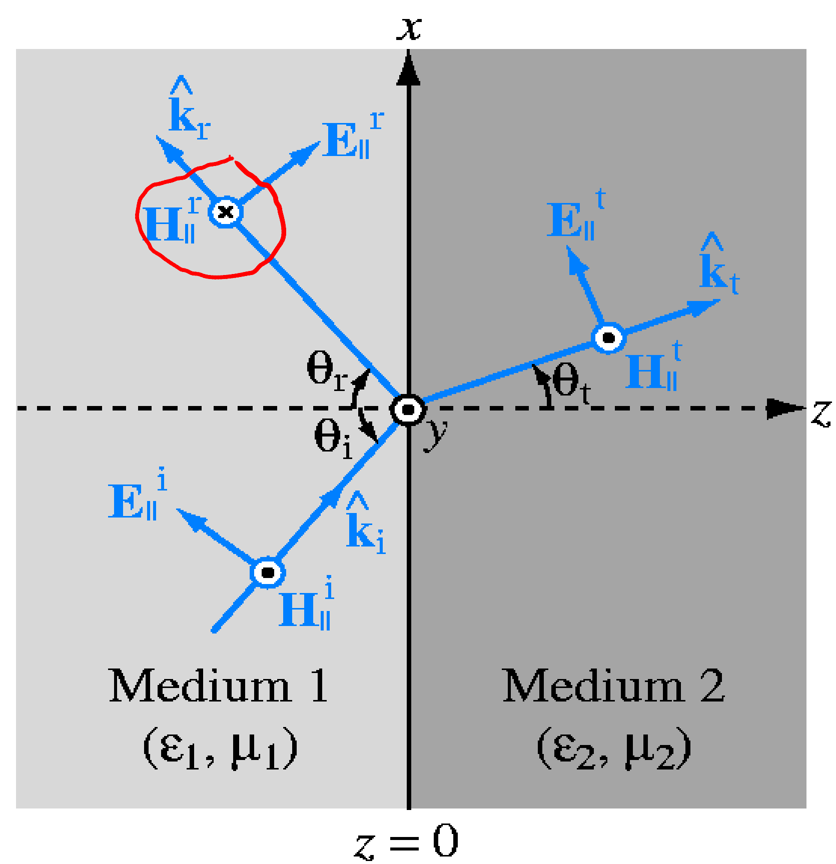
$$\left|\vec{E}_i\right| = E_o e^{-j\vec{k}_i \bullet \vec{r}}$$

The plane of incidence (in this case) is defined by the x-axis (the interface) and $\vec{k}_i$



(a) Perpendicular polarization

$\vec{E}$ perpendicular to the plane of incidence



(b) Parallel polarization

$\vec{E}$ parallel to the plane of incidence

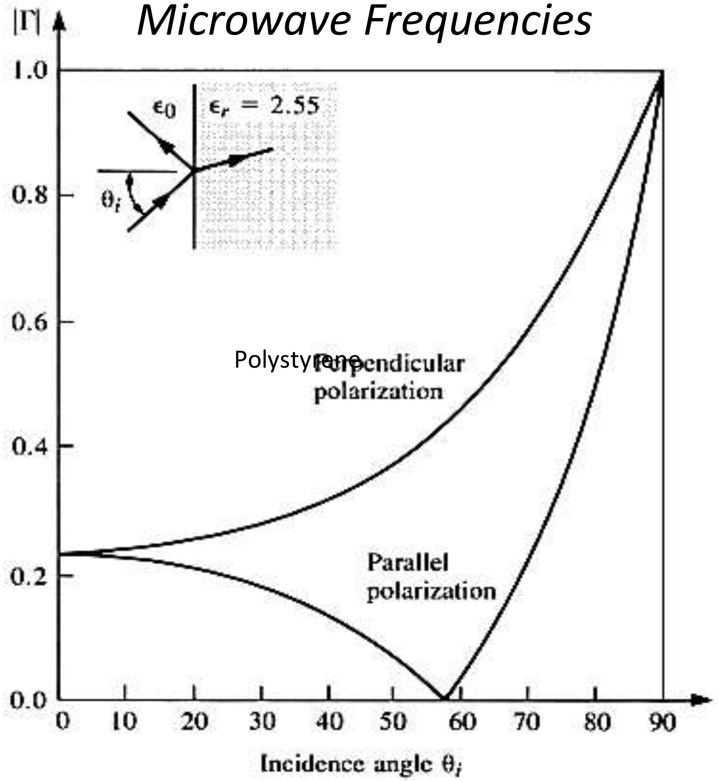
Some Polarization Notation

At Interface	
Perpendicular	Parallel
$E_{\perp}$	E_{II}
s (senkrecht)	p (parallel)
σ	π
In the Atmosphere	
V (E-field Vertical)	H (E field Horizontal)

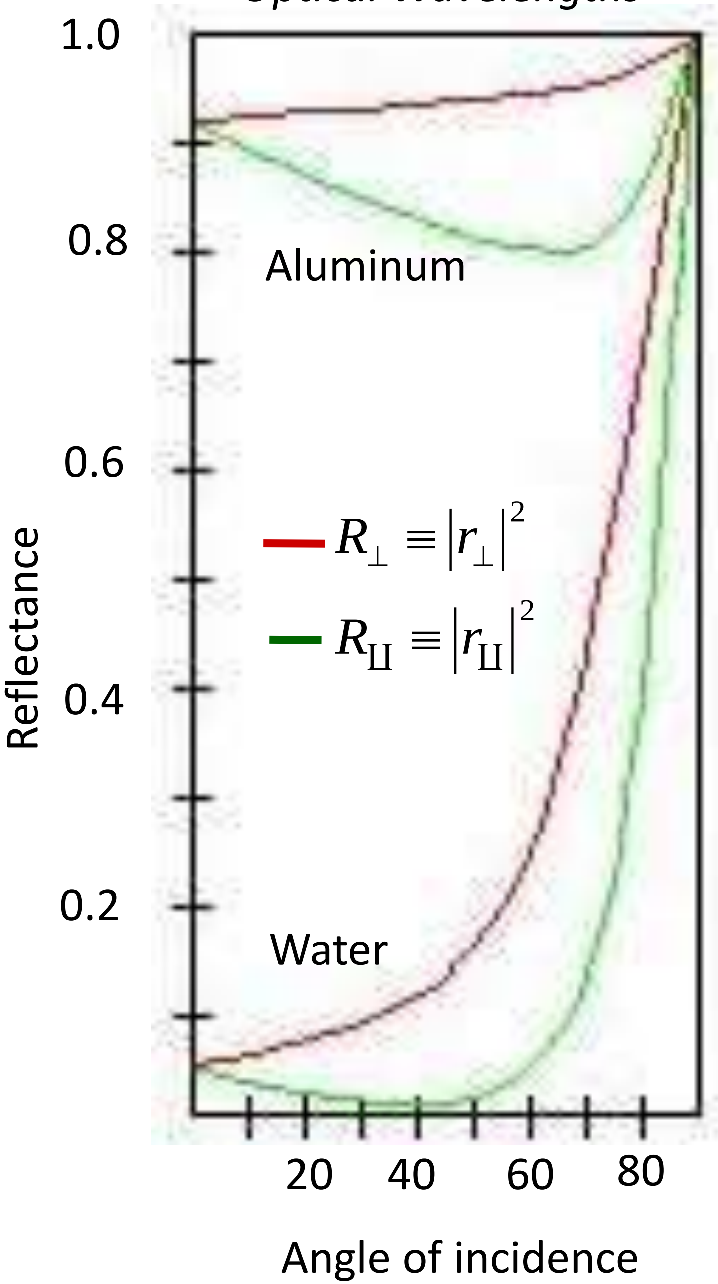
Some Examples…

http://www.ub.es/java optics/index-en.html

Microwave Frequencies



Optical Wavelengths



8-4 (finish) and 8-5

Lecture 35

Snell's

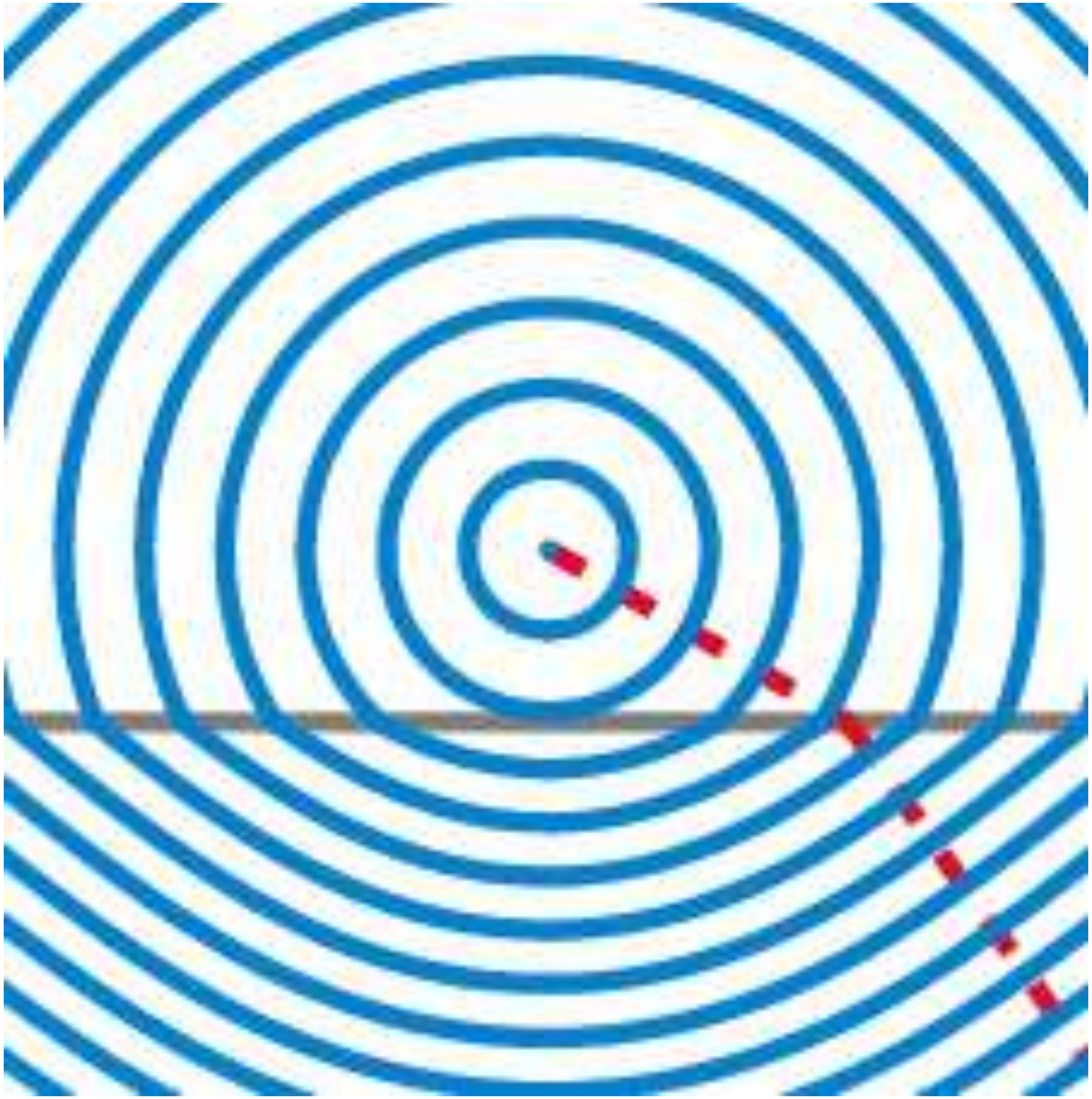
…applies to **all** waves

- Least Time
- Photons and Conservation of Momentum
- Waves and Boundary Conditions

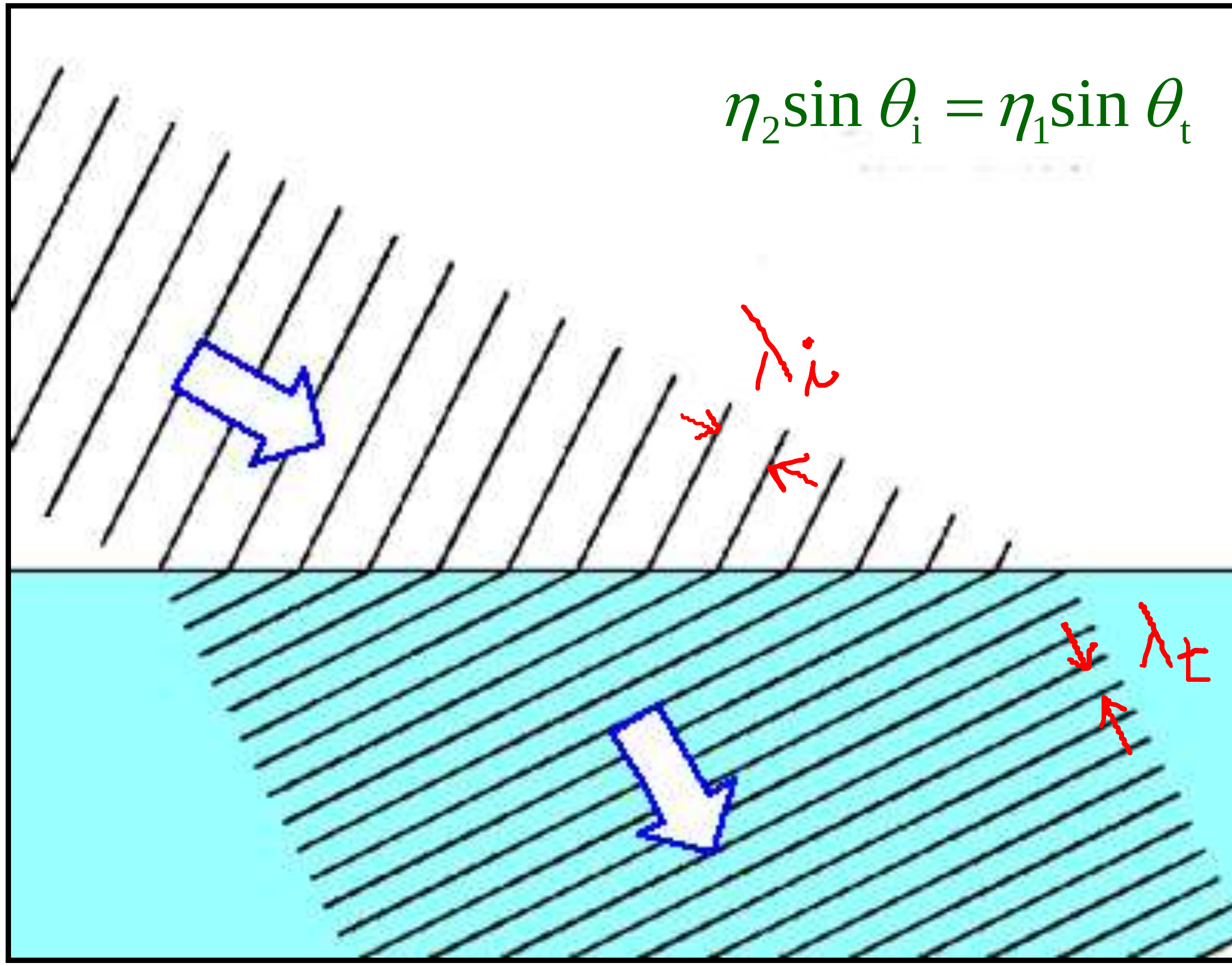
$$\eta_2 \sin\theta_{\mathrm{i}} = \eta_1 \sin\theta_{\mathrm{t}}$$

Can also show: $\theta_{\mathrm{r}} = \theta_i$

Refraction !!
(and Reflection)



Snell's Law & Total Internal Reflection hold true for both Parallel and Perpendicular Polarizations

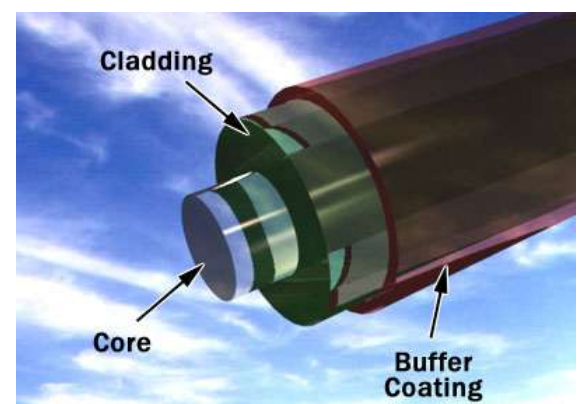


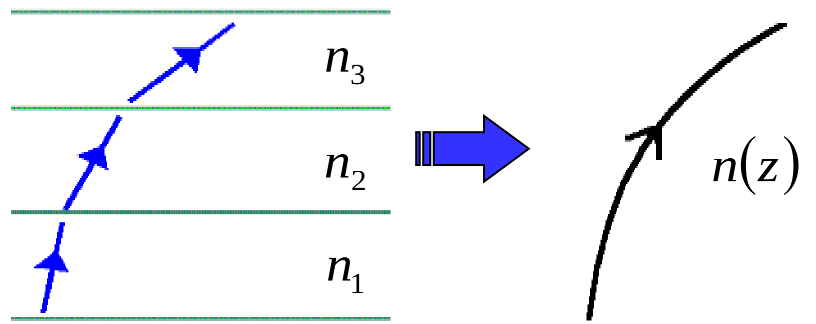
http://www.lon-capa.org/~mmp/kap25/Snell/app.htm

Total Internal Reflection

Fiber
Prism
Etc.

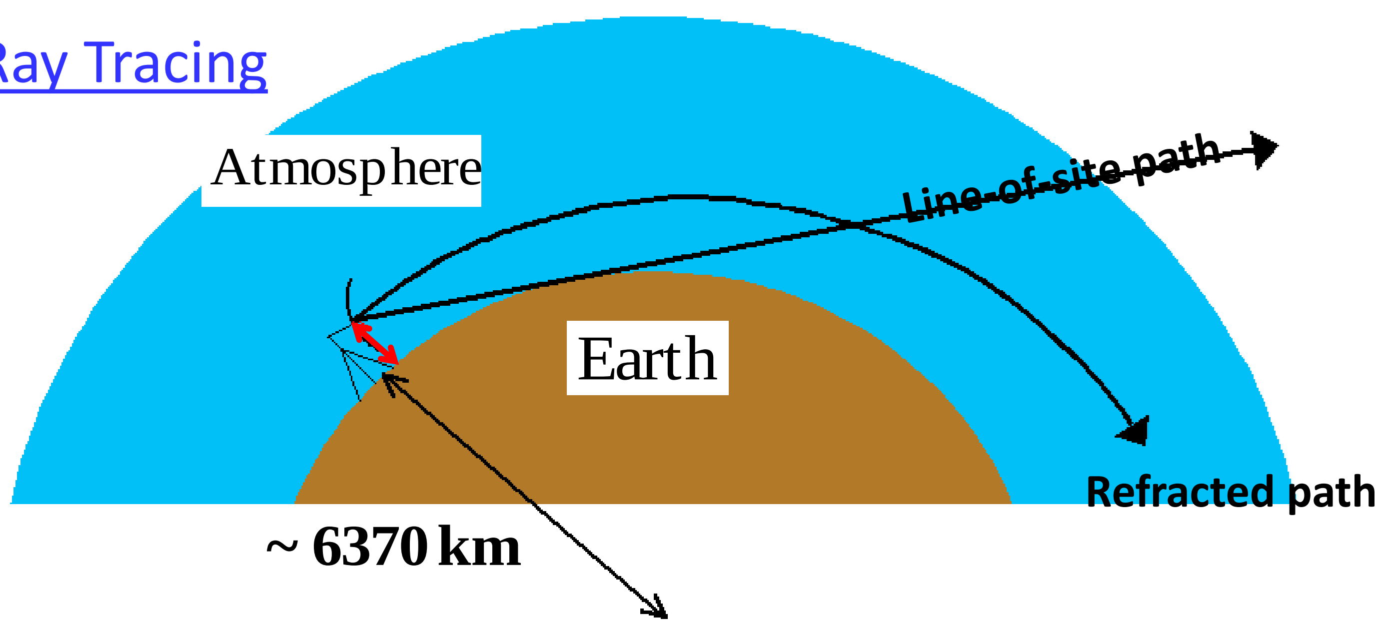
$$\theta_i \geq \sin^{-1}\frac{\eta_1}{\eta_2}$$



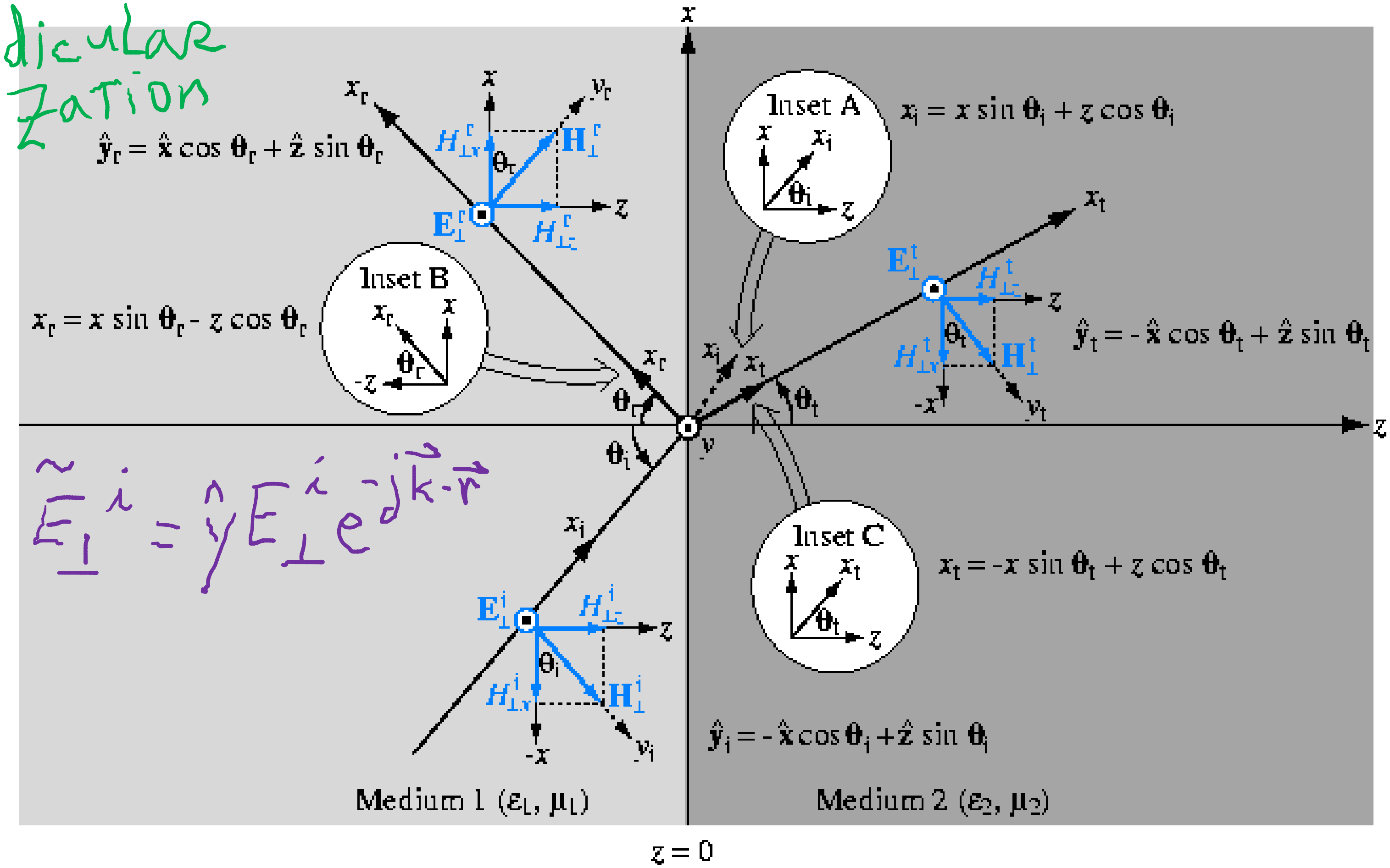


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Ray Tracing



Perpendicular Polarization



$$\tilde{E}_\perp^i = \hat{y} E_\perp^i e^{-j\vec{k}\cdot\vec{r}}$$

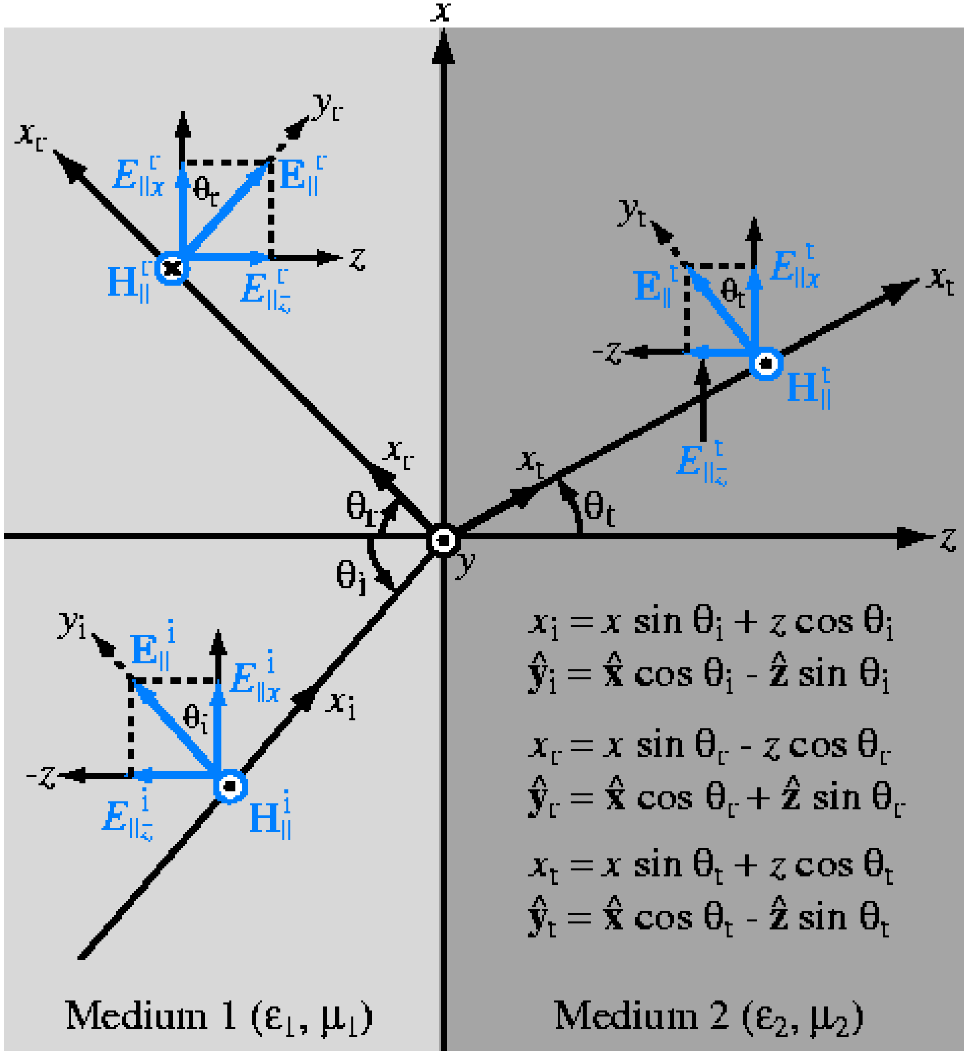
$$\frac{E_\perp^r}{E_\perp^i} = \Gamma_\perp$$

and

$$\frac{E_\perp^t}{E_\perp^i} = \tau_\perp$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$



$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Parallel Polarization

For $\parallel$-polarization, $r_{\parallel} = 0$

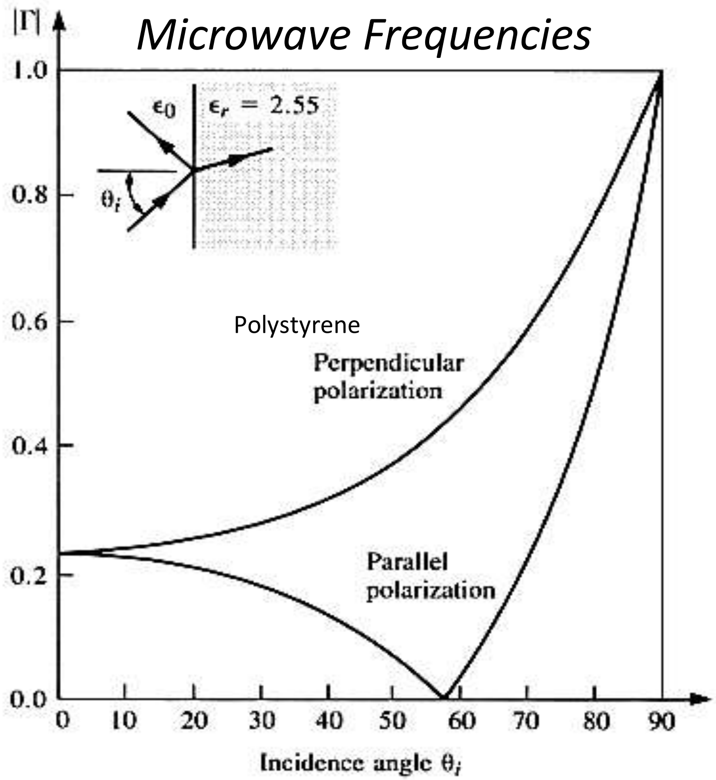
$$\text{if} \quad \theta_i = \theta_b = \tan^{-1}\sqrt{\frac{\eta_1}{\eta_2}}$$

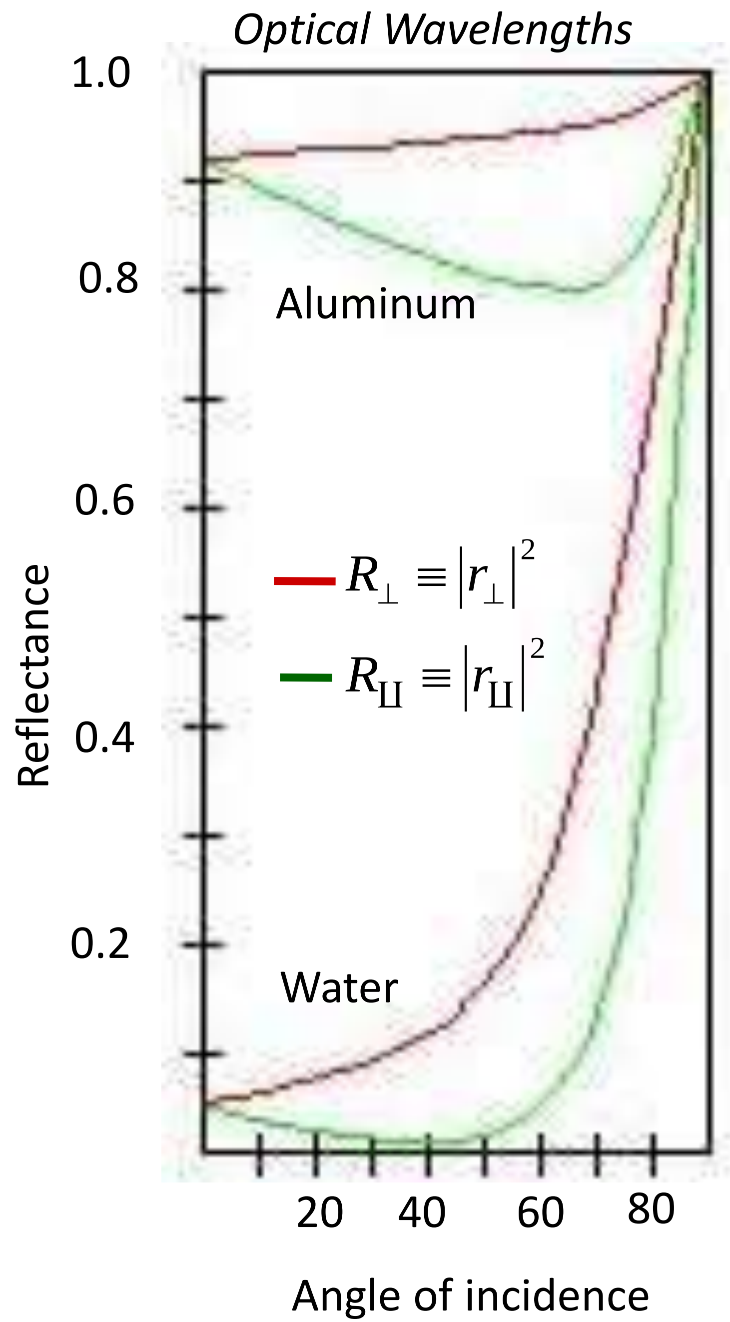
Brewster.... (only for ll-pol.)

If both regions are dielectric

Some Examples...

http://www.ub.es/java optics/index-en.html





Expressions for Γ, τ, R, and T for wave incidence from a medium with intrinsic impedance η_1 onto a medium with intrinsic impedance η_2. Angles θ_i and θ_t are the angles of incidence and transmission, respectively.

Property	Normal Incidence $\theta_i = \theta_t = 0$	Perpendicular Polarization	Parallel Polarization
Reflection coefficient	$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_\perp = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\Gamma_\parallel = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Transmission coefficient	$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_\perp = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\tau_\parallel = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Relation of Γ to τ	$\tau = 1 + \Gamma$	$\tau_\perp = 1 + \Gamma_\perp$	$\tau_\parallel = (1 + \Gamma_\parallel) \dfrac{\cos\theta_i}{\cos\theta_t}$
Reflectivity	$R = \lvert\Gamma\rvert^2$	$R_\perp = \lvert\Gamma_\perp\rvert^2$	$R_\parallel = \lvert\Gamma_\parallel\rvert^2$
Transmissivity	$T = \lvert\tau\rvert^2 \left(\dfrac{\eta_1}{\eta_2}\right)$	$T_\perp = \lvert\tau_\perp\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$	$T_\parallel = \lvert\tau_\parallel\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$
Relation of R to T	$T = 1 - R$	$T_\perp = 1 - R_\perp$	$T_\parallel = 1 - R_\parallel$

Notes: (1) $\sin\theta_t = \sqrt{\mu_1\varepsilon_1/\mu_2\varepsilon_2}\,\sin\theta_i$; (2) $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$; (3) $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$; (4) for nonmagnetic media, $\eta_2/\eta_1 = n_1/n_2$.

Transmitted and Reflected Powers

Note: gotta take into account projected areas...

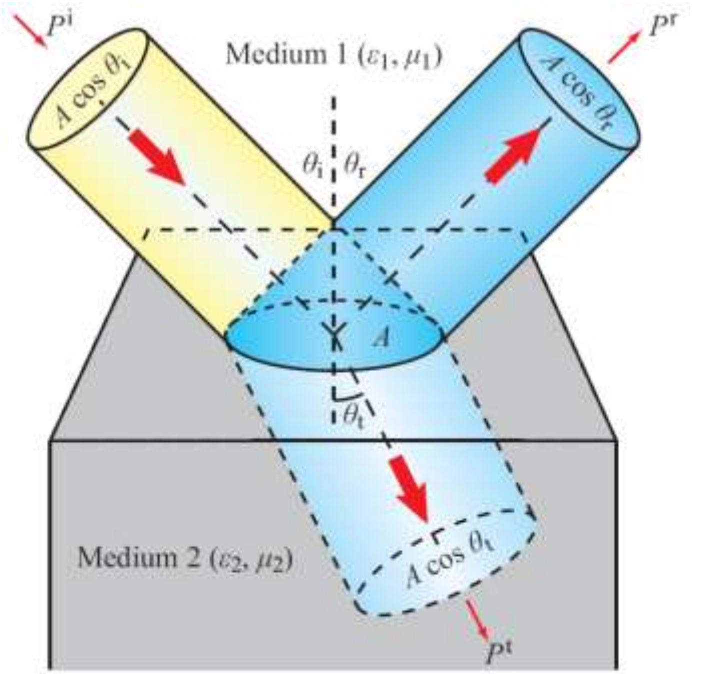
Power Reflectivity and Transmissivity

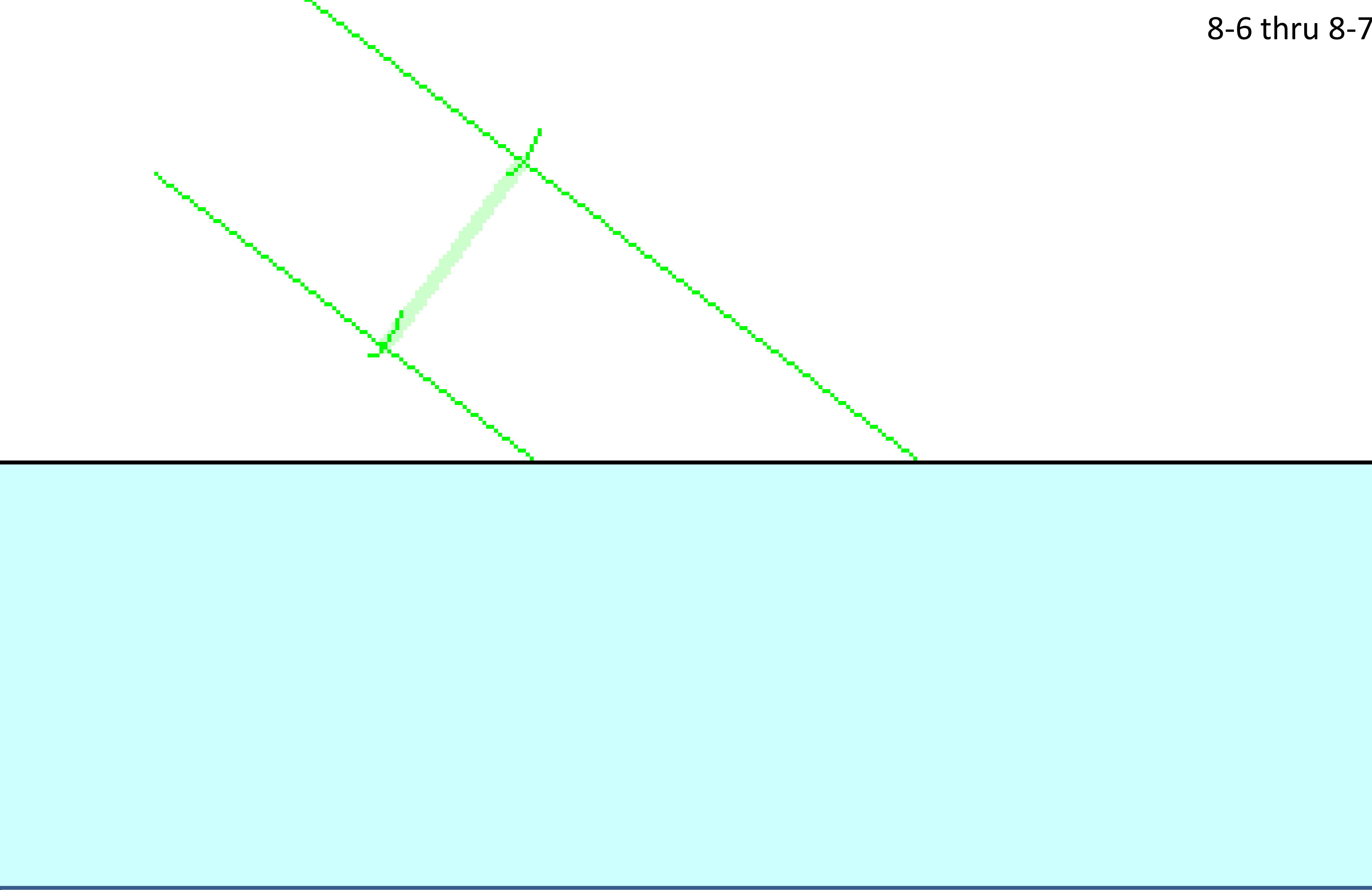
$$T_\perp = \frac{P^{\mathrm{t}}_\perp}{P^{\mathrm{i}}_\perp} = \frac{|E^{\mathrm{t}}_{\perp 0}|^2}{|E^{\mathrm{i}}_{\perp 0}|^2} \frac{\eta_1}{\eta_2} \frac{A\cos\theta_{\mathrm{t}}}{A\cos\theta_{\mathrm{i}}}$$

$$= |\tau_\perp|^2 \left(\frac{\eta_1\cos\theta_{\mathrm{t}}}{\eta_2\cos\theta_{\mathrm{i}}}\right), \qquad (8.79a)$$

$$T_\parallel = \frac{P^{\mathrm{t}}_\parallel}{P^{\mathrm{i}}_\parallel} = |\tau_\parallel|^2 \left(\frac{\eta_1\cos\theta_{\mathrm{t}}}{\eta_2\cos\theta_{\mathrm{i}}}\right). \qquad (8.79b)$$

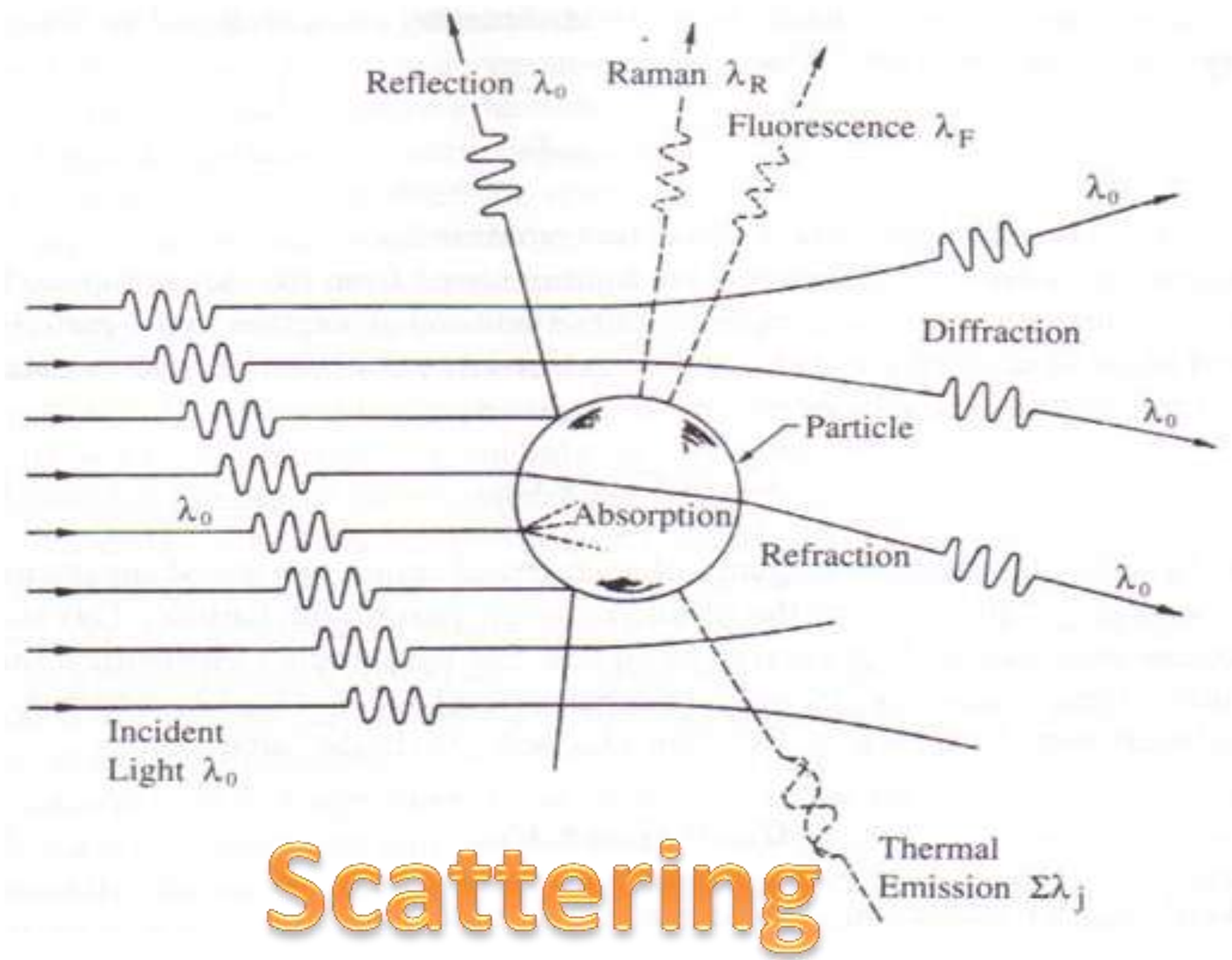
The incident, reflected, and transmitted waves do not have to obey any such laws as conservation of electric field, conservation of magnetic field, or conservation of power density, but they do have to obey the law of conservation of power.

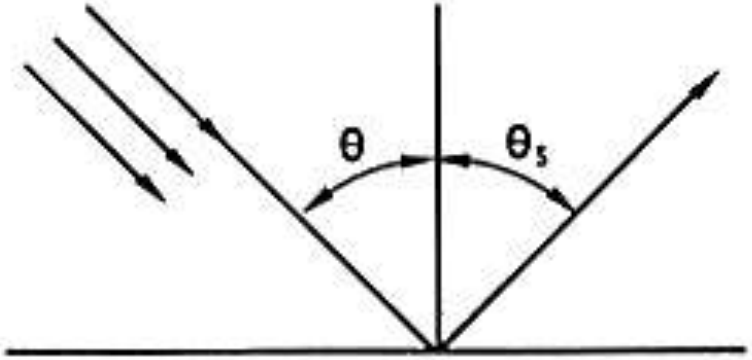




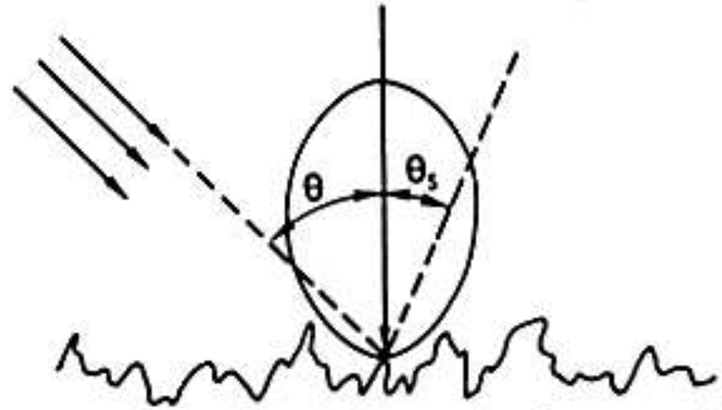
What direction will it go once it gets here? ☺

http://www.guardian.co.uk/environment/2009/oct/28/stealth-wind-turbines-radar

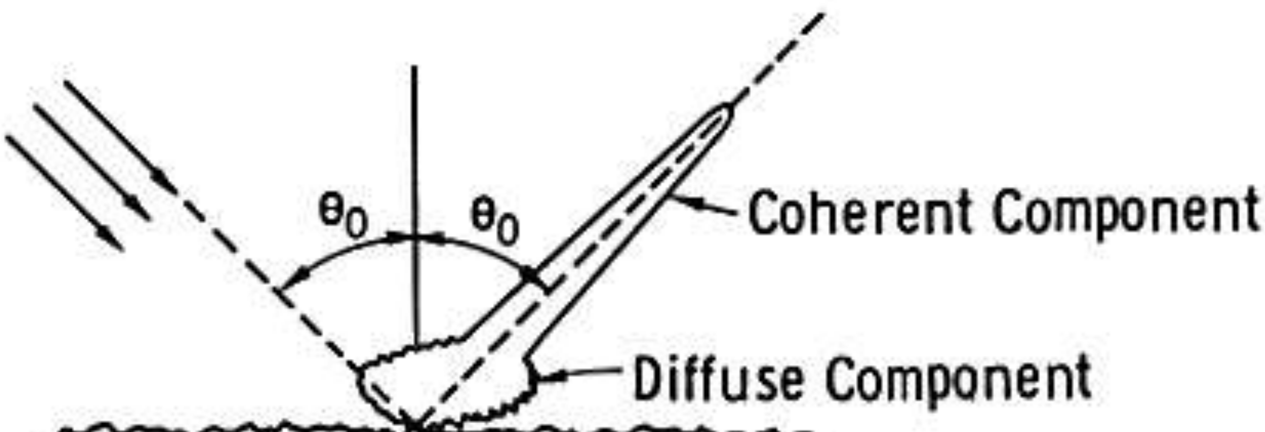




Reflected Power is Entirely Coherent and $\theta_s = \theta$, Scattering Pattern is a Delta Function

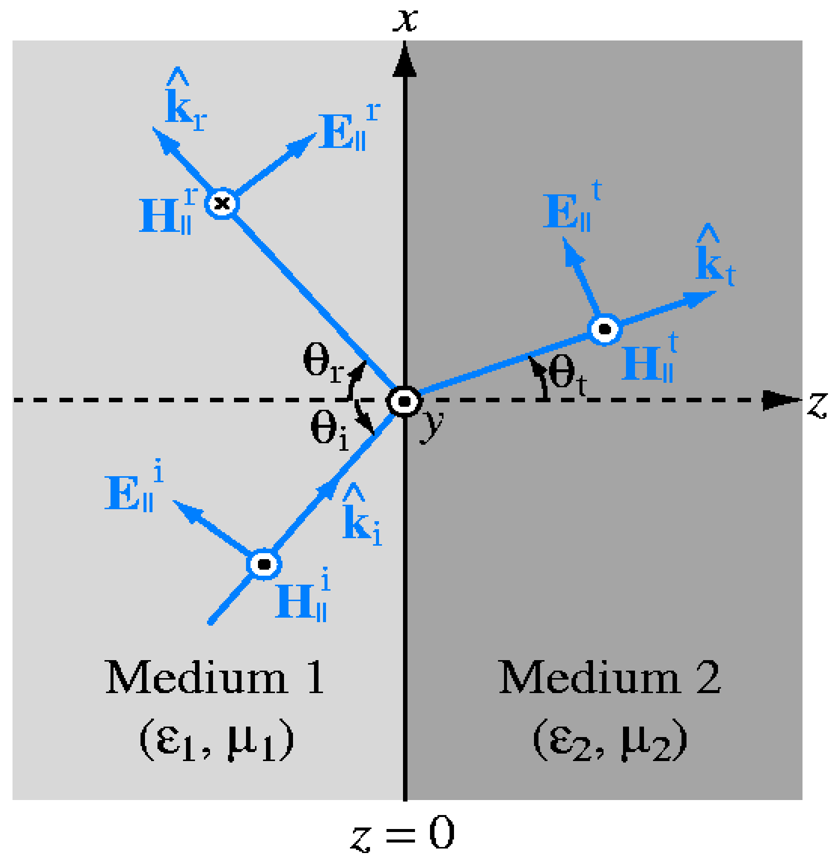


Scattering Pattern is Composed Entirely of Diffuse Component. For Lambertian Surface, $\sigma^\circ(\theta, \theta_s) = \sigma_0^\circ \cos\theta \cos\theta_s$

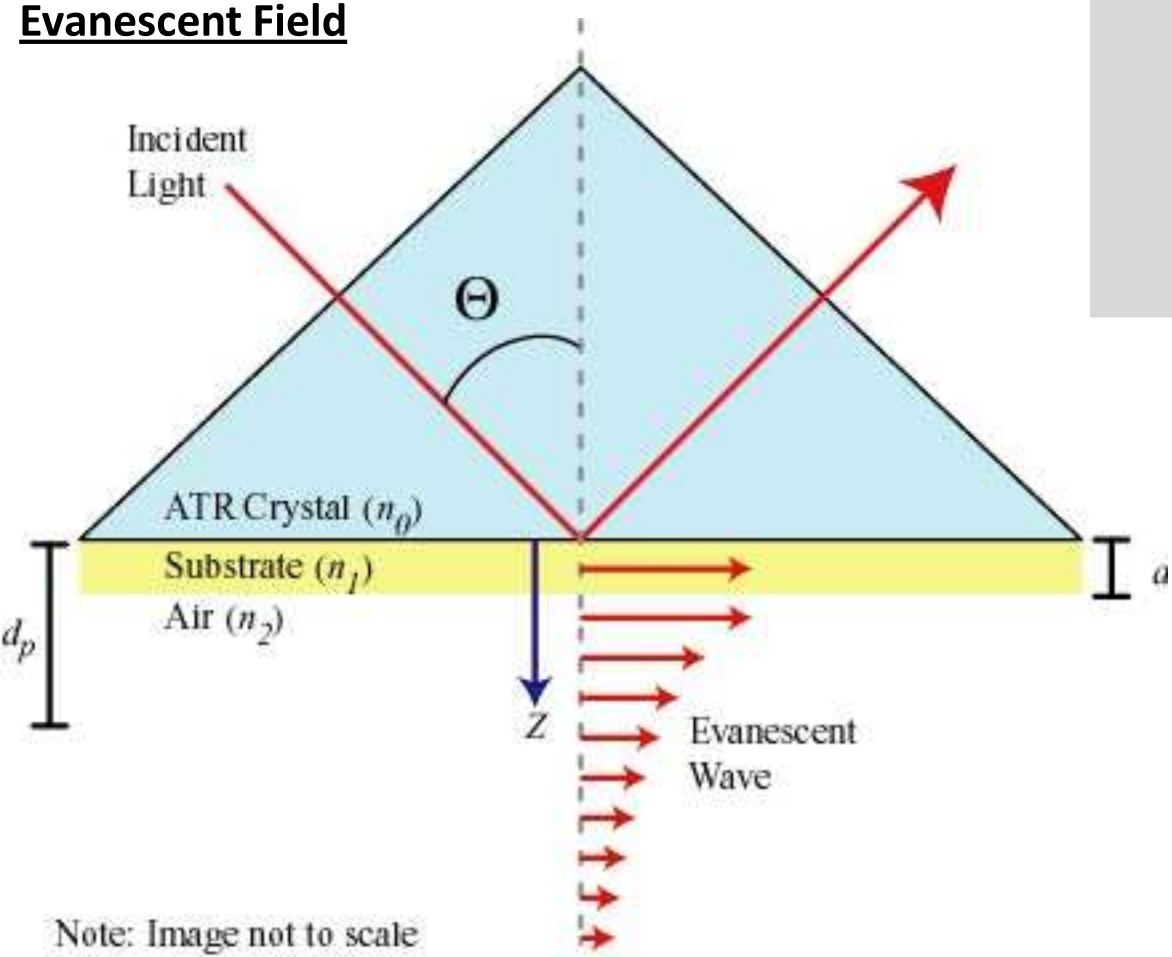


Scattering Pattern Consists of Large Coherent Component and Small Diffuse Component

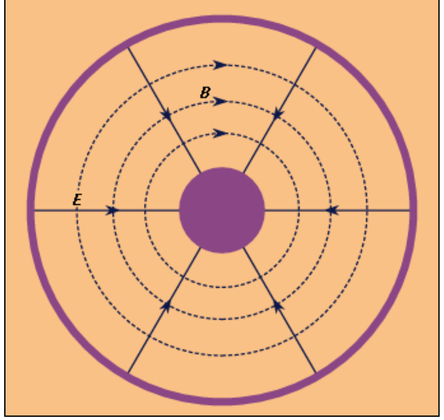
$$\theta_i \geq \sin^{-1} \frac{n_1}{n_2}$$

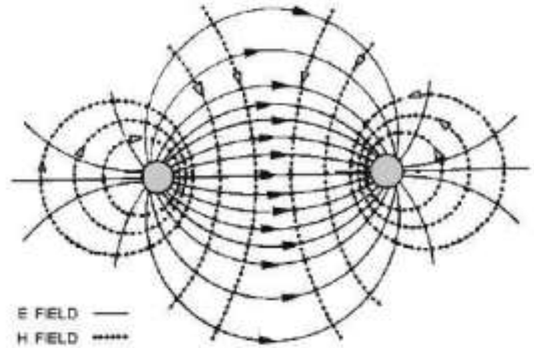


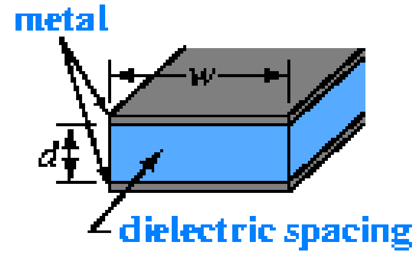
Evanescent Field



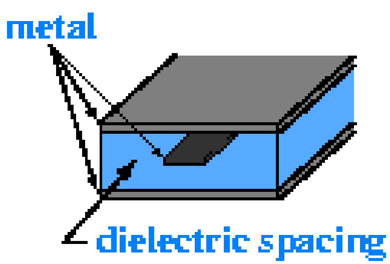
http://www.olympusmicro.com/primer/java/tirf/evaintensity/



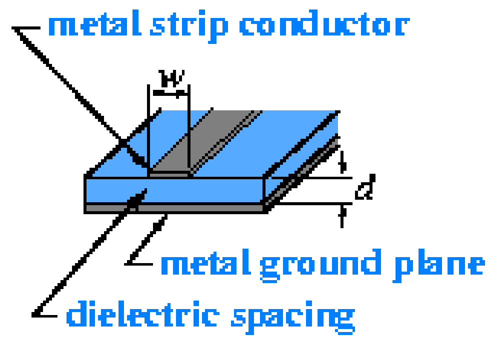




(c) Parallel-plate line

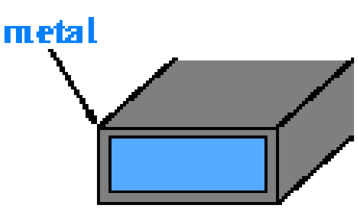


(d) Strip line

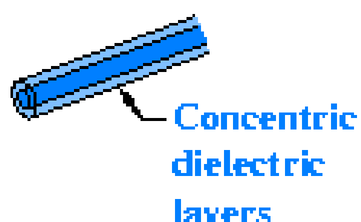


(e) Microstrip line

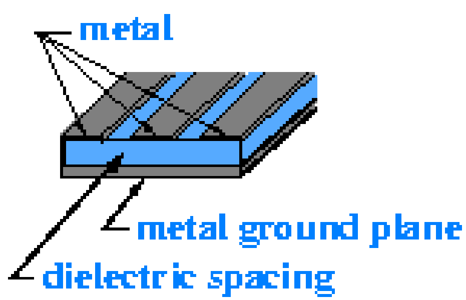
TEM Transmission Lines



(f) Rectangular waveguide

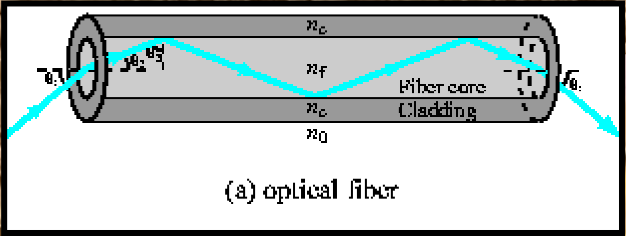


(g) Optical fiber

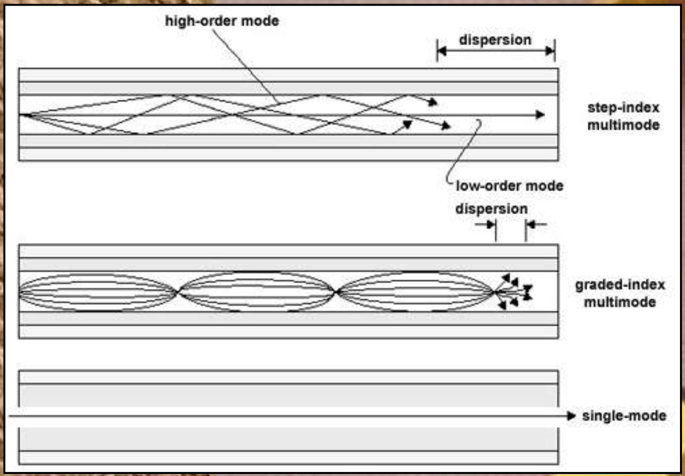


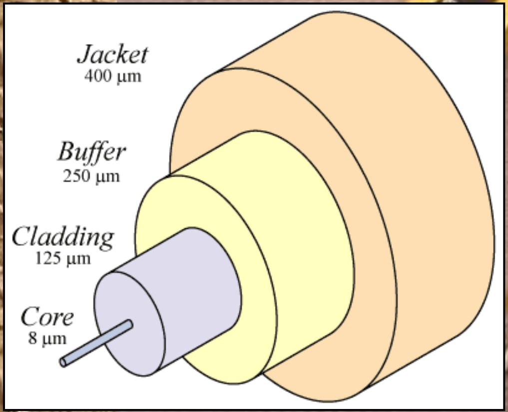
(h) Coplanar waveguide

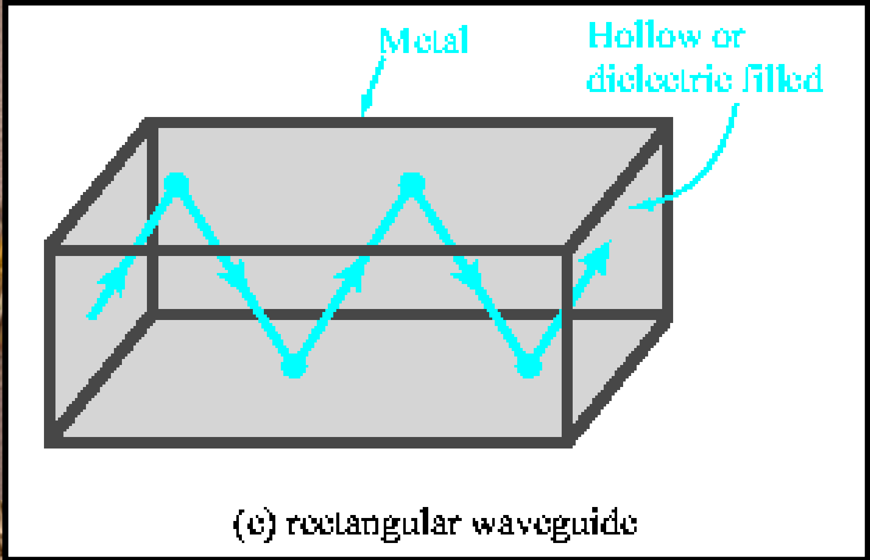
Higher Order Transmission Lines



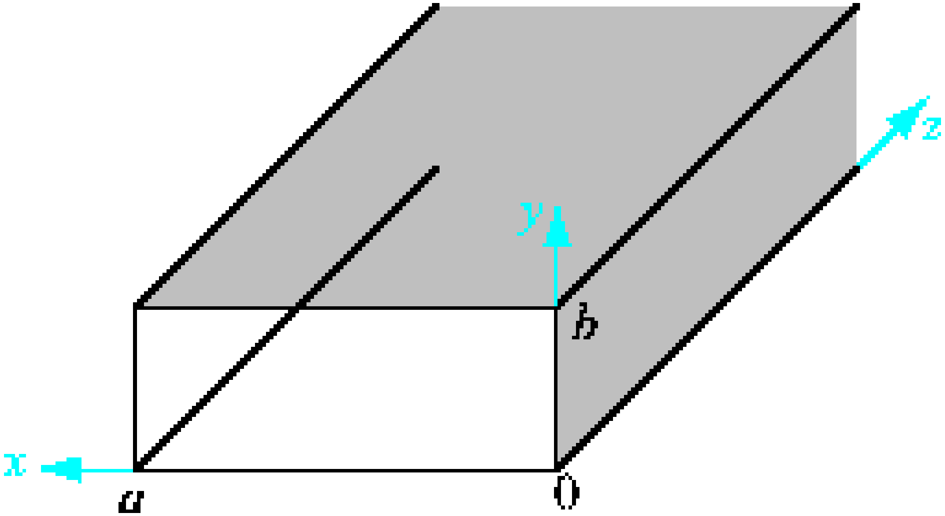
(a) optical fiber

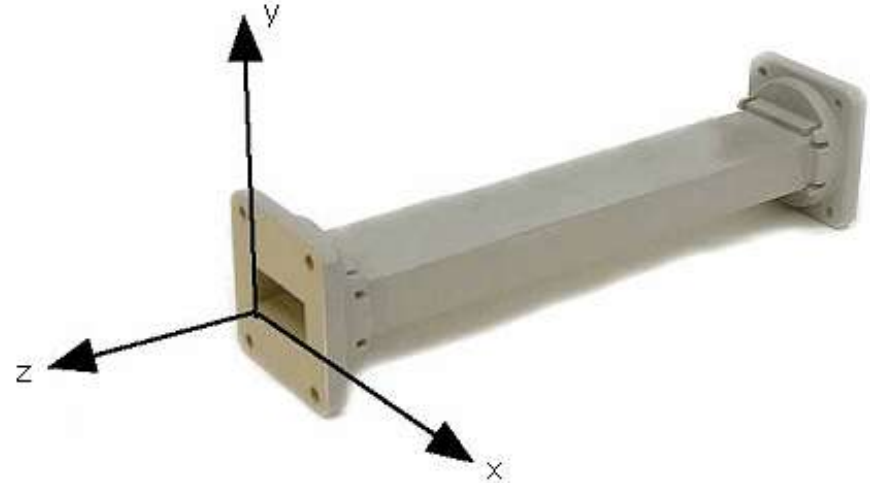




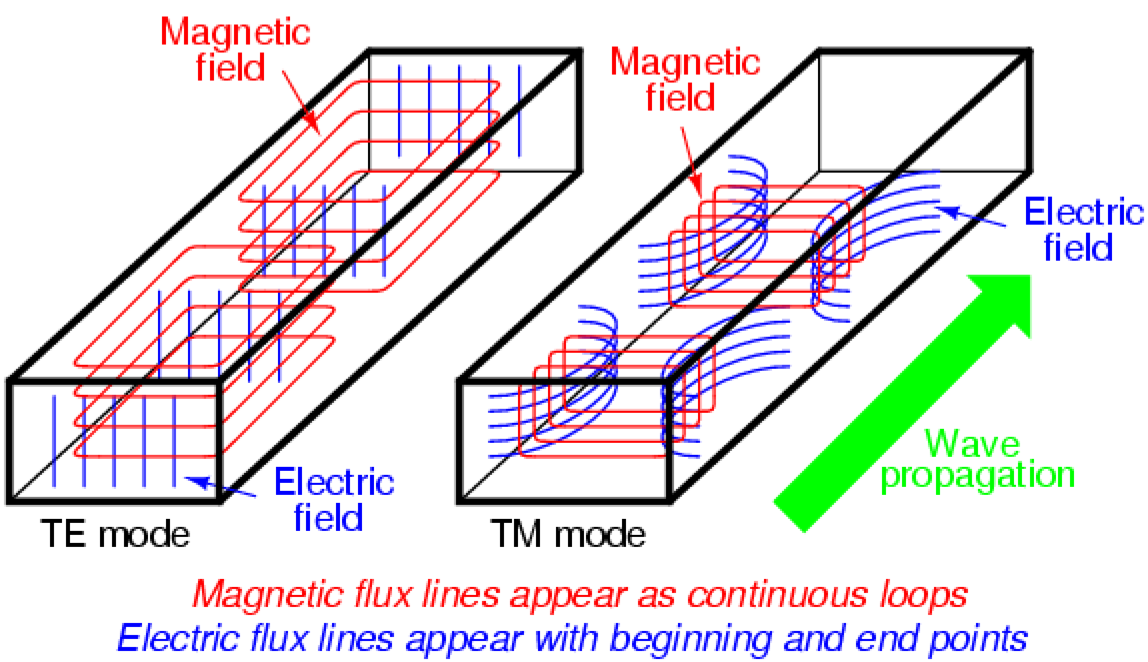


(c) rectangular waveguide





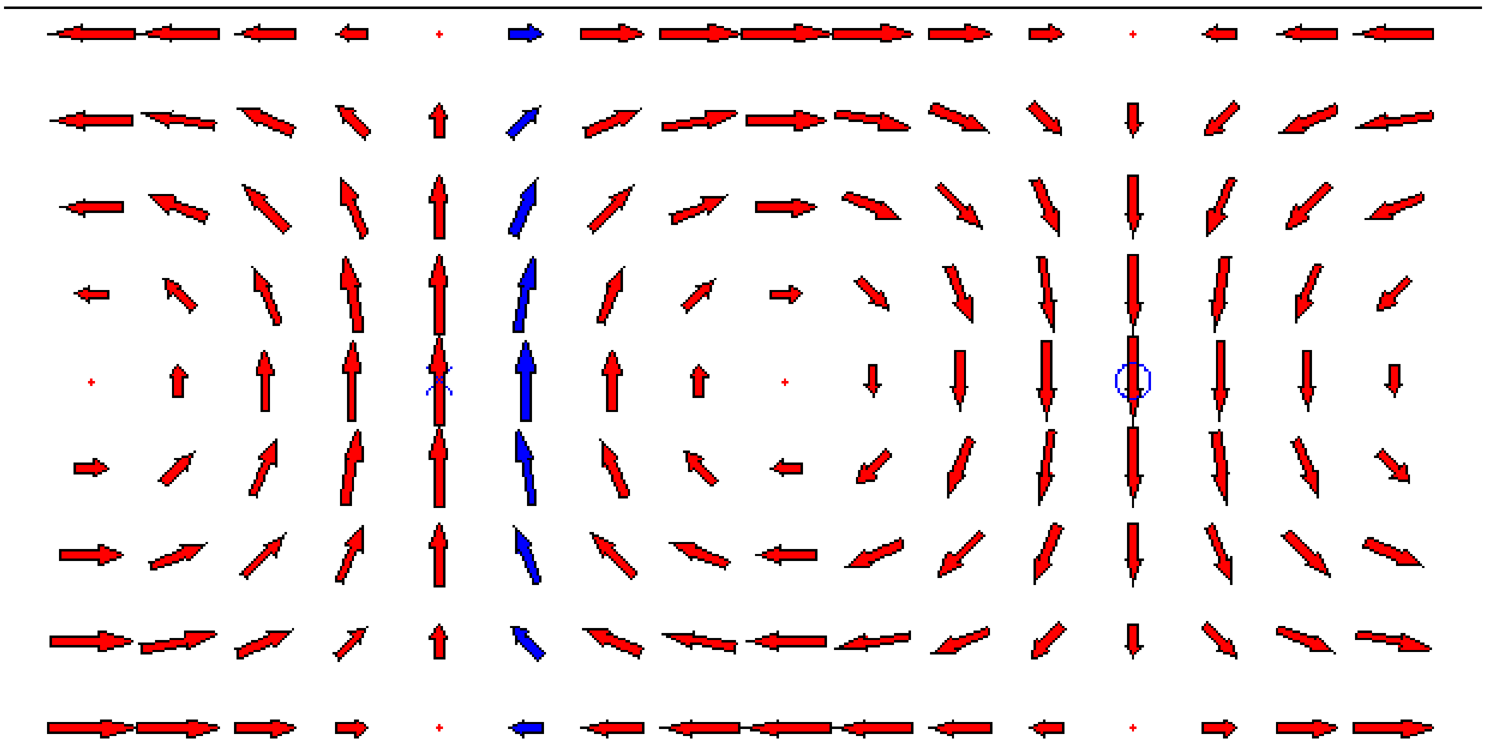
Boundary Conditions !!!



Modes!!!

http://cetaweb.mit.edu/6.630/movies/index.htm

http://www.temf.de/index.php?id=61&L=1



This is a side view of the magnetic field of a TE_{01} mode in a rectangular wave guide.

$$\eta \equiv \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_0}{\sqrt{\epsilon_R}}$$

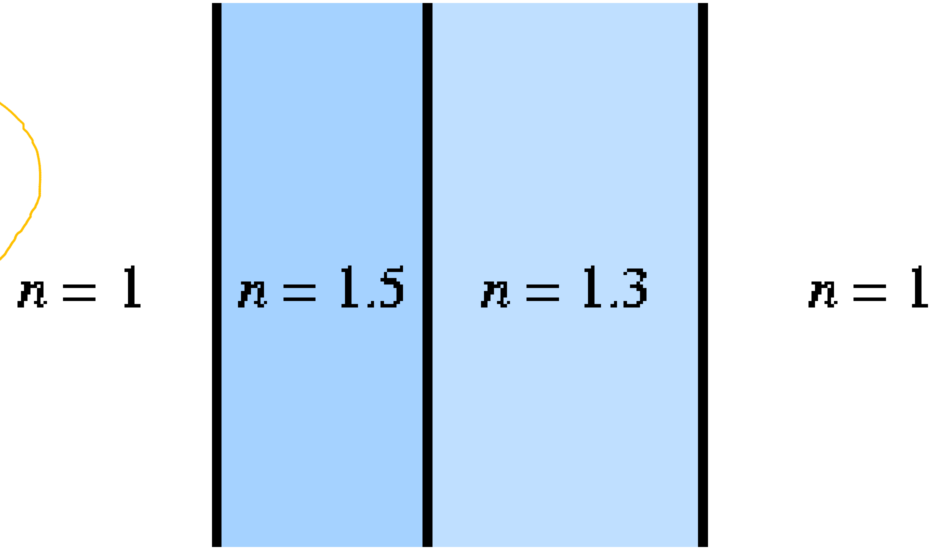
for $\mu = \mu_0$ and $\epsilon = \epsilon_R \epsilon_0$

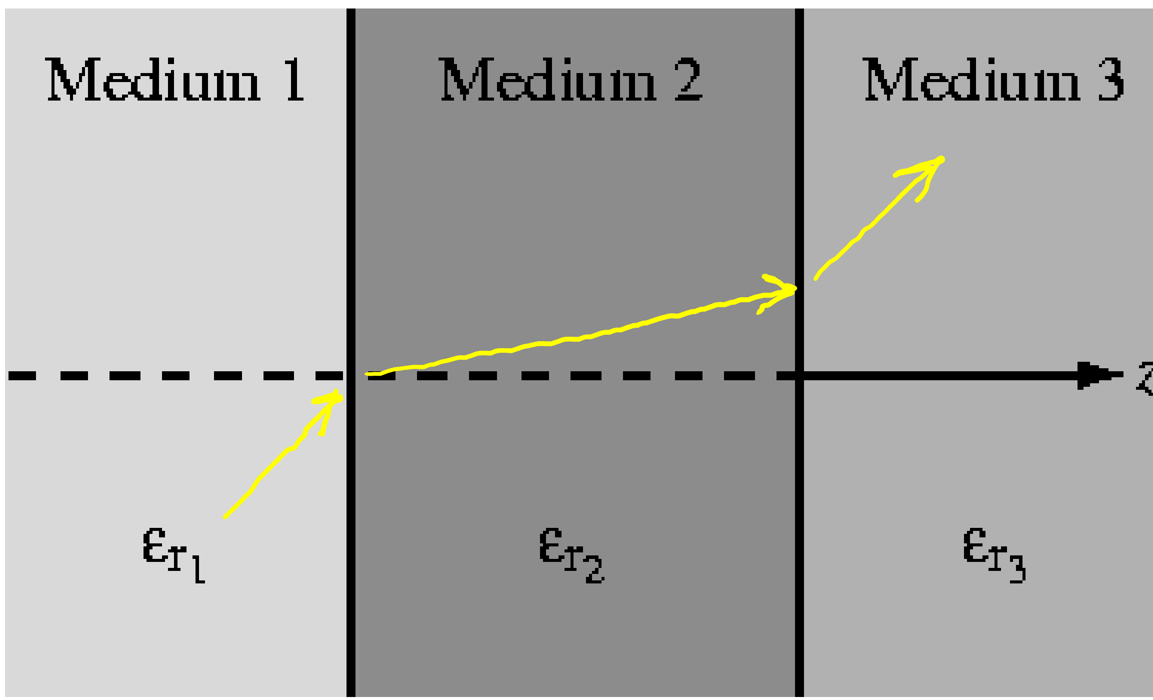
Impedance

Relative permittivity

index of Refraction

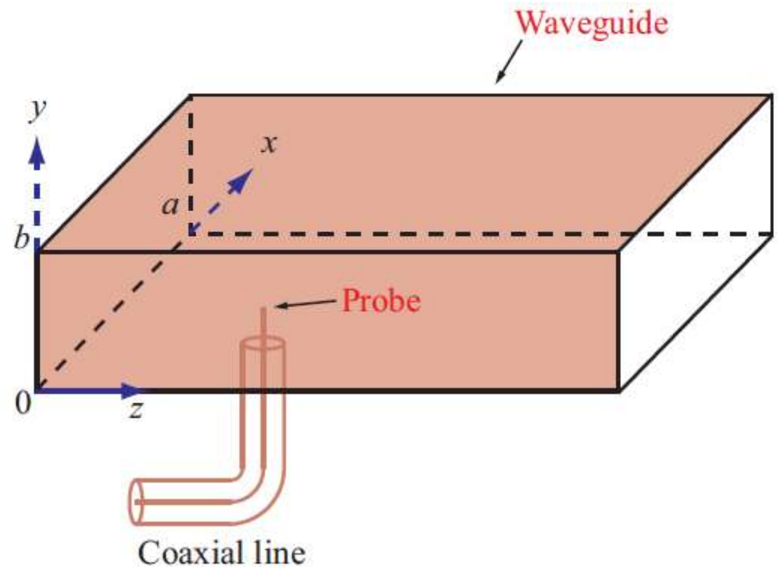
$$n = \sqrt{\epsilon_R}$$



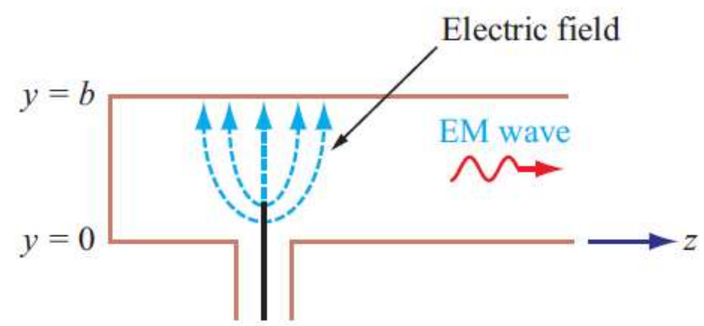


Coax-to-Waveguide Connection

An extended section of the inner conductor of a coaxial cable can serve to couple energy into a waveguide or from the waveguide

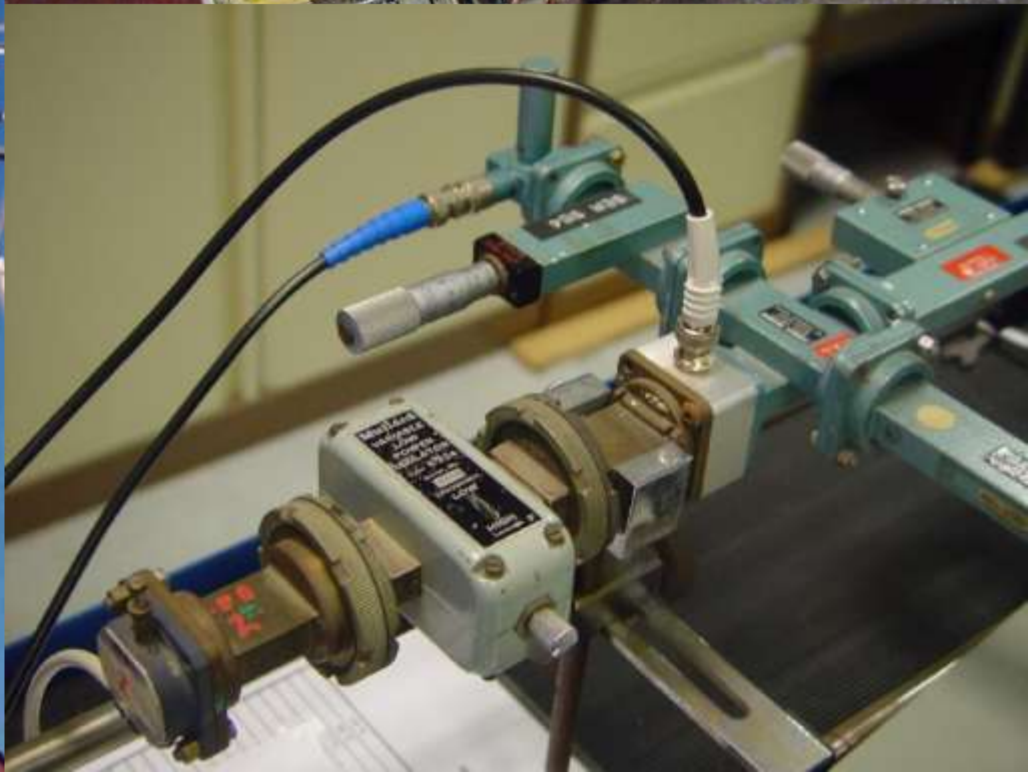


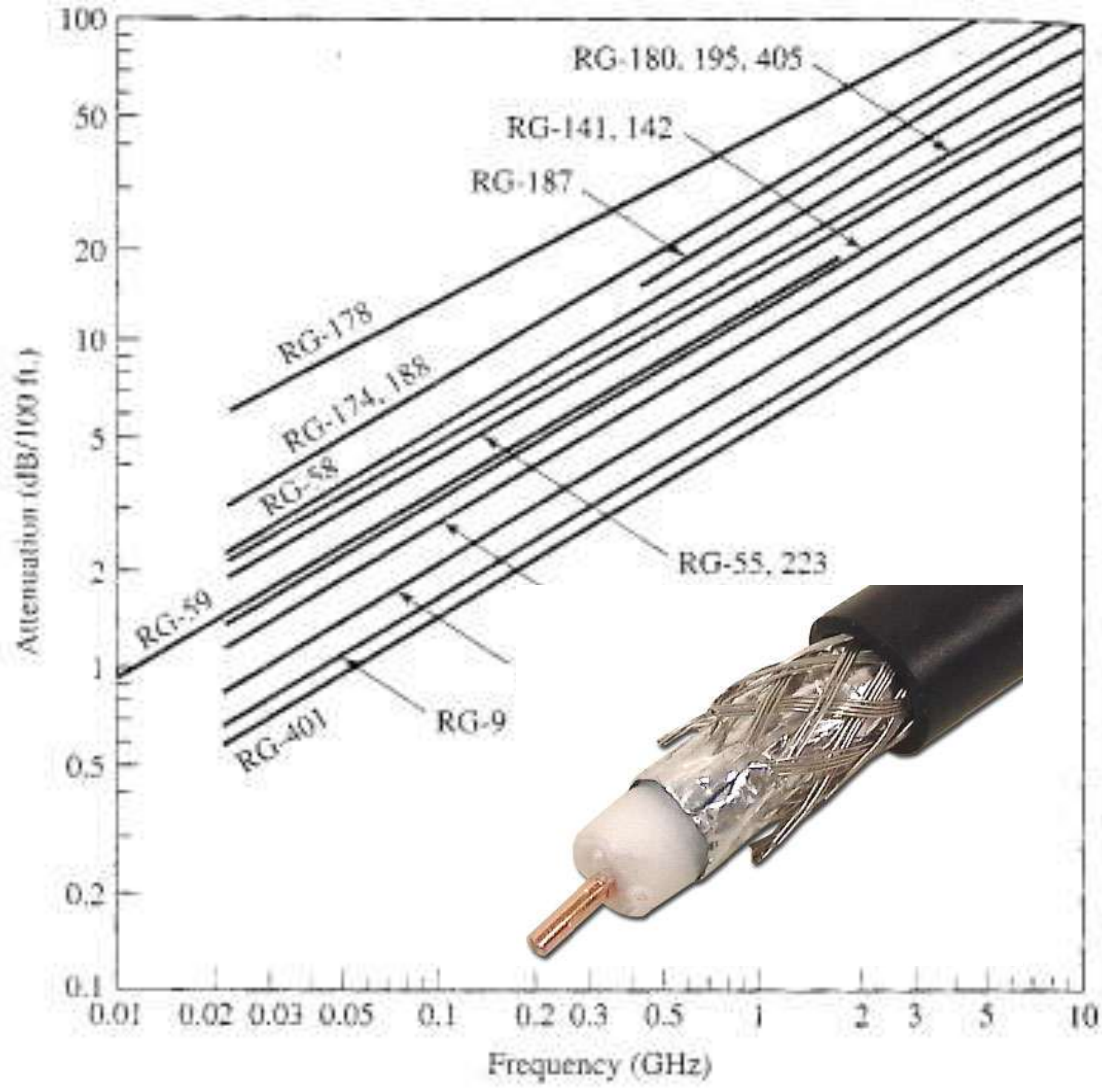
(a) Coax-to-waveguide coupler



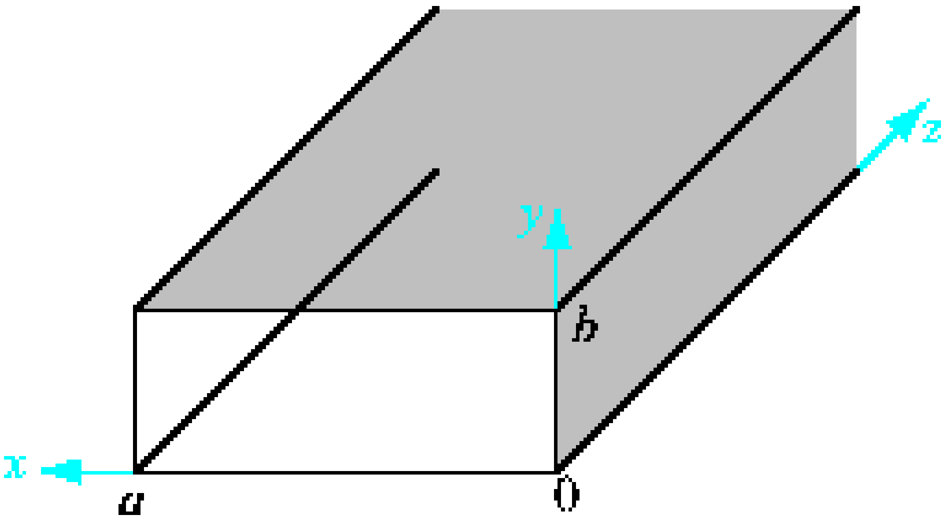
(b) Cross-sectional view at $x = a/2$

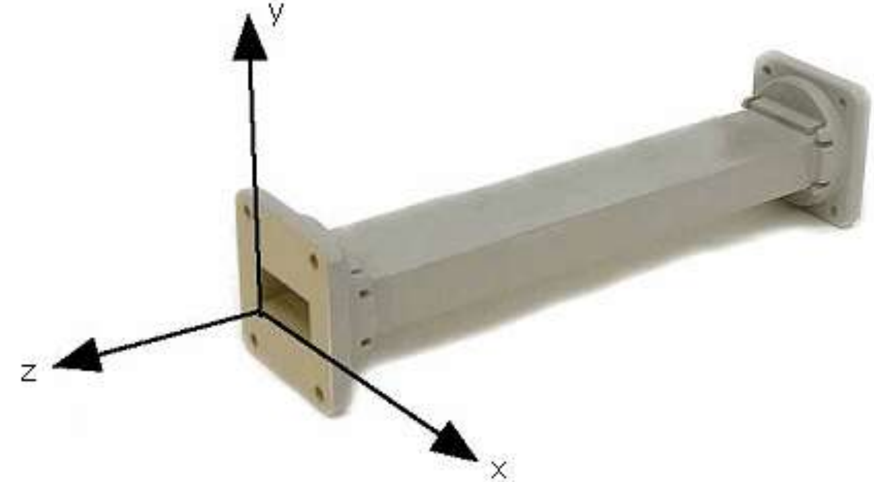
Figure 8-21: The inner conductor of a coaxial cable can excite an EM wave in the waveguide.



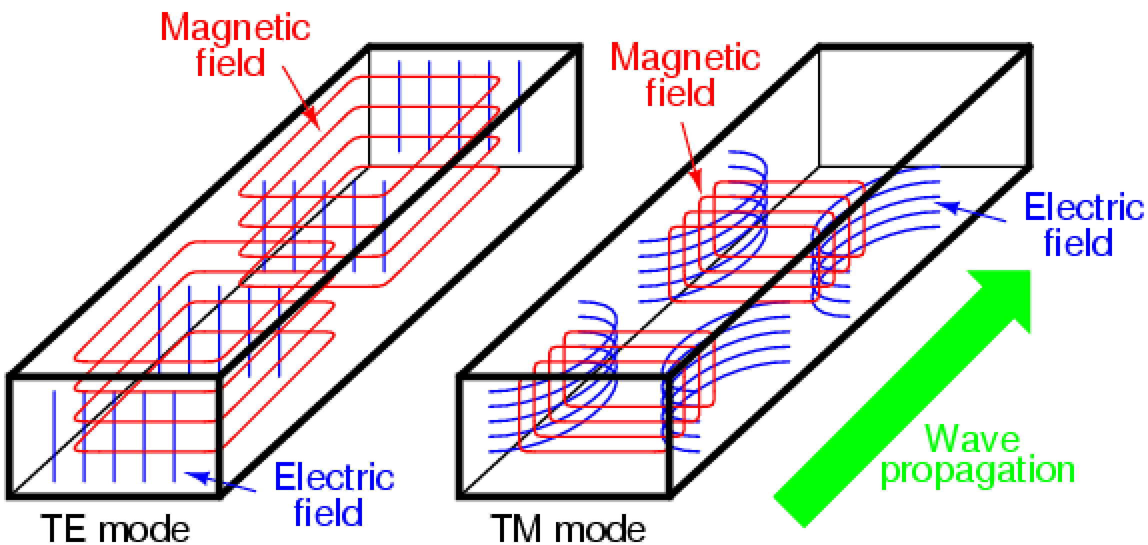


RG-180, 195, 405
RG-141, 142
RG-187
RG-178
RG-174, 188
RG-58
RG-59
RG-55, 223
RG-9
RG-401
Attenuation (dB/100 ft.)
Frequency (GHz)
100
50
20
10
5
2
1
0.5
0.2
0.1
0.01
0.02
0.03
0.05
0.1
0.2
0.3
0.5
1
2
3
5
10





Boundary Conditions !!!

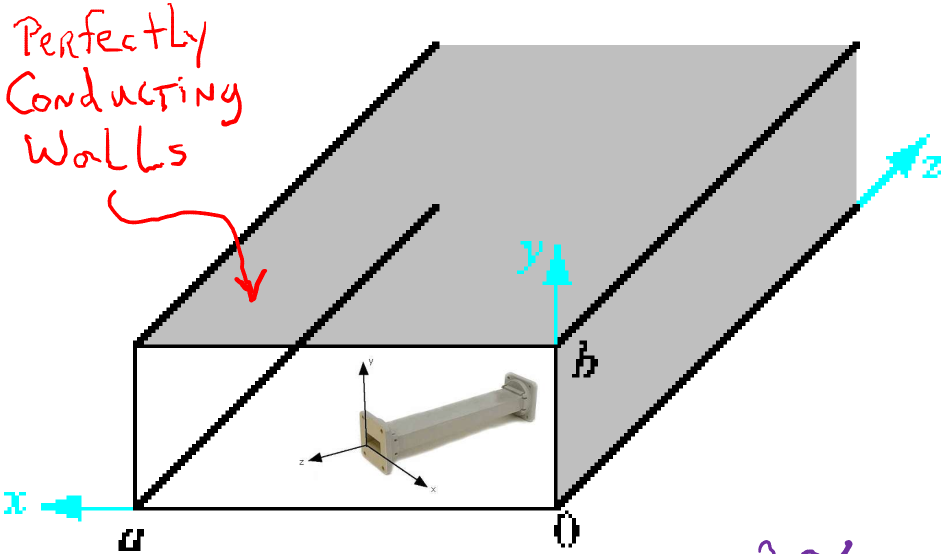


Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

Modes!!!

http://cetaweb.mit.edu/6.630/movies/index.htm

http://www.temf.de/index.php?id=61&L=1



$$\boxed{\begin{aligned}\nabla\cdot\tilde{\mathbf{E}}&=0,\\ \nabla\times\tilde{\mathbf{E}}&=-j\omega\mu\tilde{\mathbf{H}},\\ \nabla\cdot\tilde{\mathbf{H}}&=0,\\ \nabla\times\tilde{\mathbf{H}}&=j\omega\varepsilon_c\tilde{\mathbf{E}}.\end{aligned}}$$

$$\hat{n}\times(\mathbf{E}_1-\mathbf{E}_2)=0$$

$$\hat{n}\times(\mathbf{H}_1-\mathbf{H}_2)=J_S$$

$$D_{1n}-D_{2n}=\rho_S$$

$$B_{1n}-B_{2n}=0$$

$$\nabla^2\tilde{E}+k^2\tilde{E}=0 \rightarrow \tilde{E}=\tilde{E}_o e^{-j\beta z}$$

B.C.'s $\rightarrow$ $\tilde{E}_o=E_x\hat{x}+E_y\hat{y}+E_z\hat{z}$

$\tilde{H}_o=H_x\hat{x}+H_y\hat{y}+H_z\hat{z}$

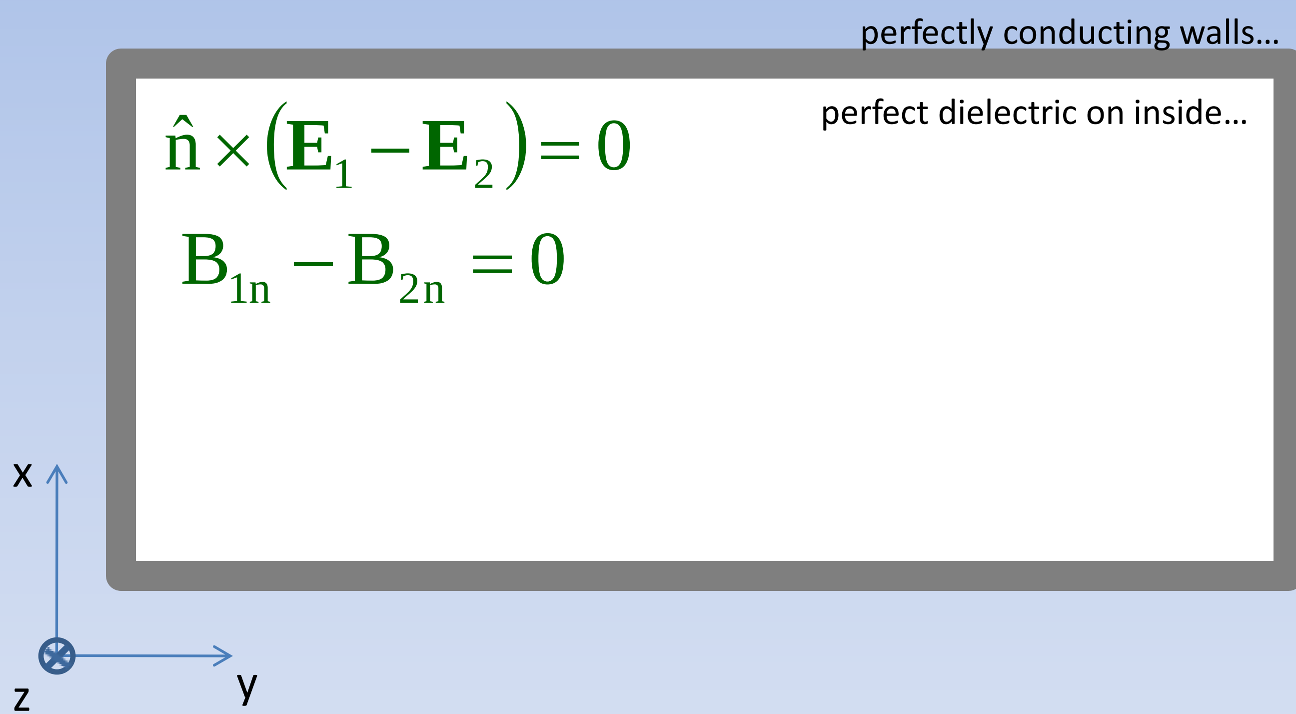
Longitudinal Components!

If $E_z=0$, $\quad \dfrac{\partial E_x}{\partial x}+\dfrac{\partial E_y}{\partial y}=0$ Gauss

If $B_z=0$, $\quad \dfrac{\partial E_y}{\partial x}-\dfrac{\partial E_x}{\partial y}=0$ Faraday

$+\ \hat{n}\times(\mathbf{E}_1-\mathbf{E}_2)=0 \Rightarrow$ No TEM in hollow metal waveguide

tangential components zero at walls



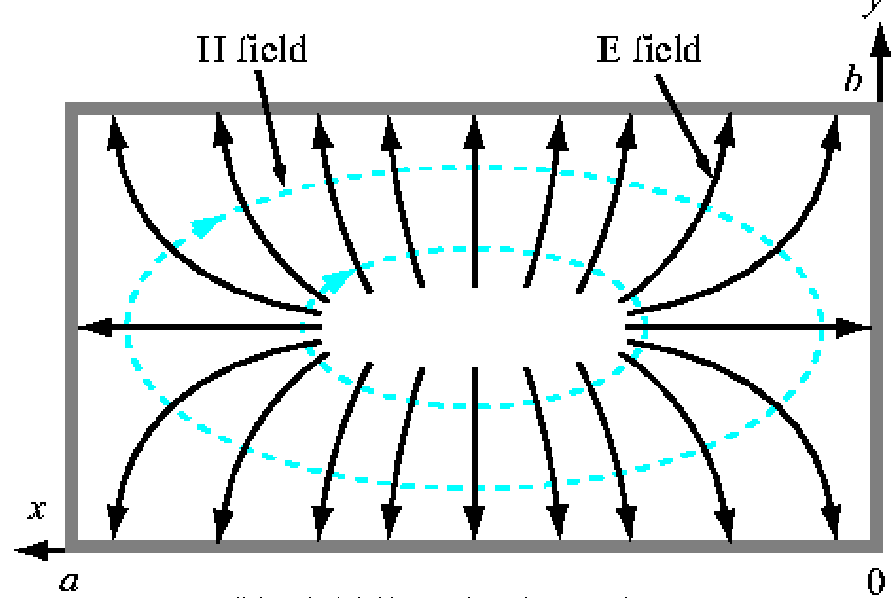
$$\hat{n} \times \left(\mathbf{E}_1 - \mathbf{E}_2\right) = 0$$

$$B_{1n} - B_{2n} = 0$$

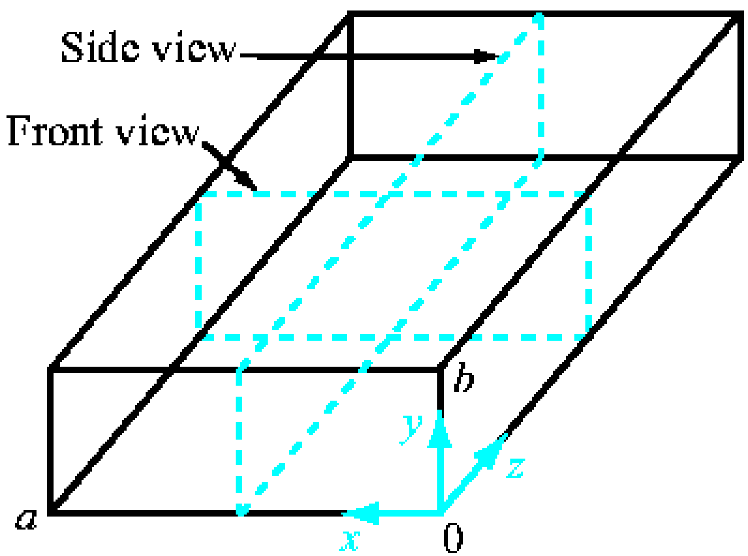
One possible mode

$\Rightarrow TM_{11}$

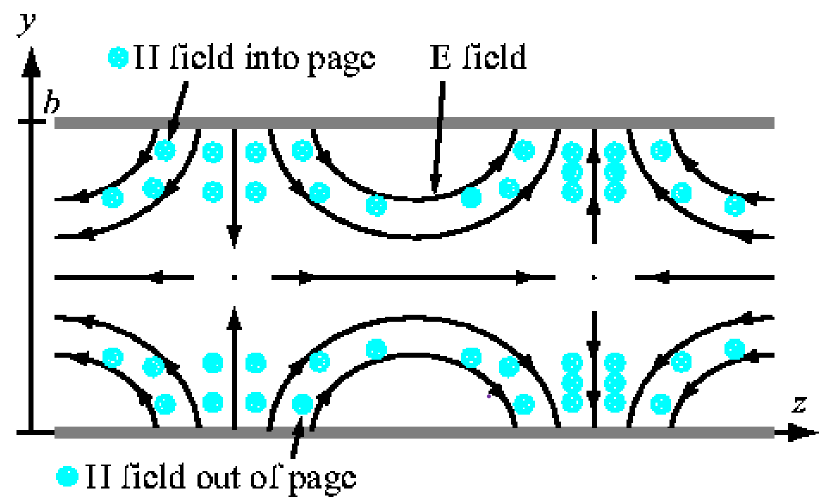
Note: more than one solution can exist at the same time!



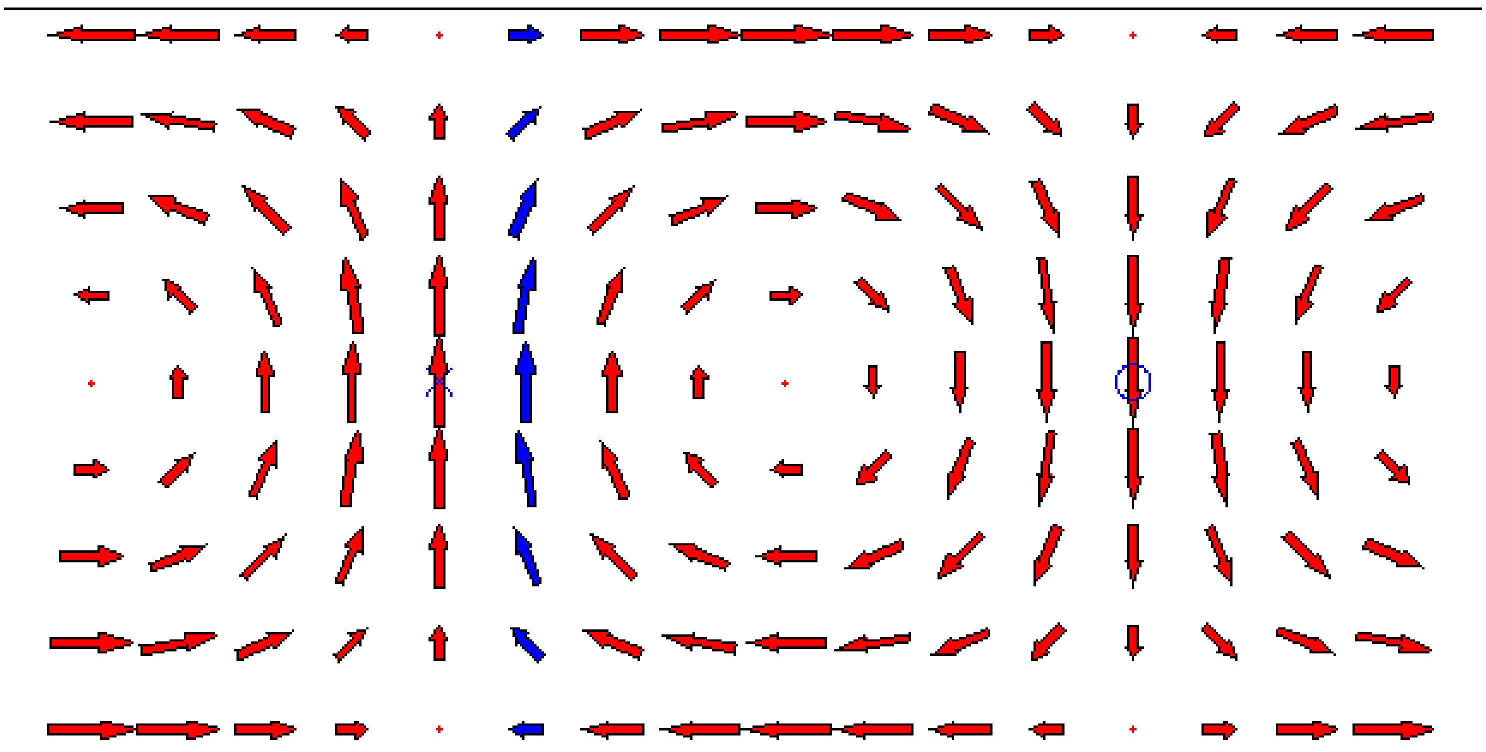
(b) Field lines for front view



(a) Cross-sectional planes



(c) Field lines for side view



This is a side view of the magnetic field of a TE_{01} mode in a rectangular wave guide.

Now, $\tilde{E} = \tilde{E}_o e^{-j\beta z}$ $\Rightarrow$ Propagate in $+z$ direction

with

$k = \omega\sqrt{\mu\epsilon}$ $\Rightarrow$ Unbounded propagation wavenumber.

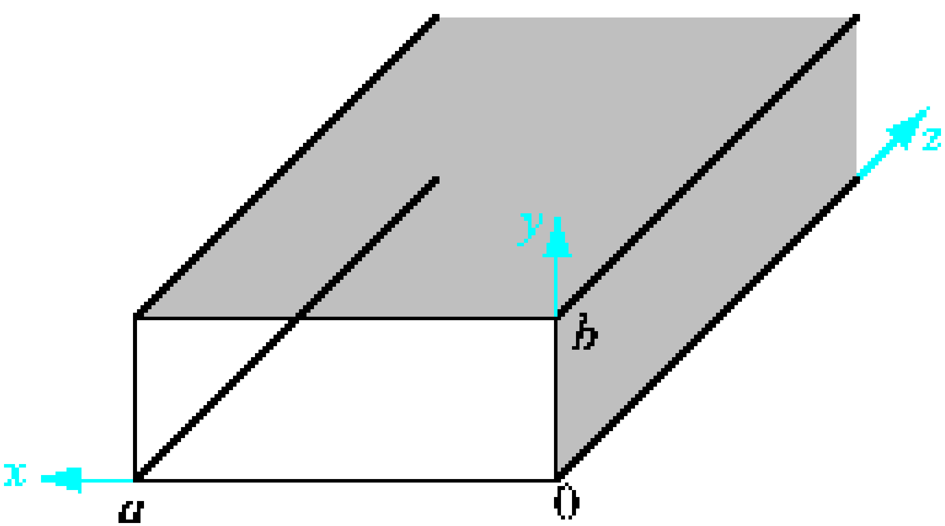
and

$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$ $\Rightarrow$ Prop. constant in waveguide

($m, n \in$ Int.; satisfy B.C.s)

each m, n represents a solution.

each soln is called a mode.



http://www.falstad.com/embox/guide.html

$$\beta = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$\tilde{E} = \tilde{E}_0 e^{-j\beta z}$$

Represents a wave iff β is real!

$$k = \omega\sqrt{\mu\epsilon} = \frac{2\pi f}{u_{p_0}} = \frac{2\pi}{\lambda_0}$$

unbounded phase speed

cut-off frequency

$$f > \frac{u_{p_0}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Note: $a > b$

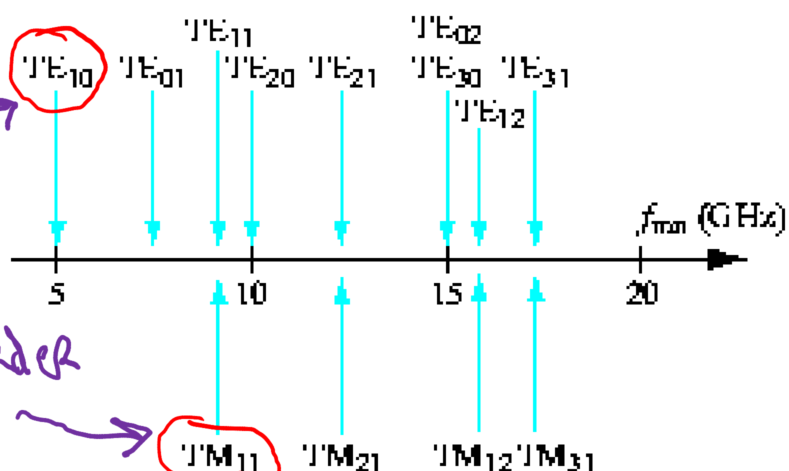
this condition can also be written $\Rightarrow$

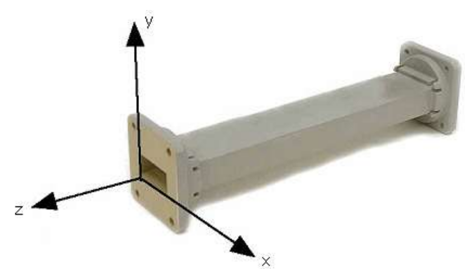
$$\frac{1}{\lambda_0^2} > \left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2$$

Lowest order TE

Dominant Modes

Lowest order TM





Cutoff frequencies for TE and TM modes
llow rectangular waveguide with $a = 3$ cm and
m (Example 8-8).

What is the cutoff frequency for the dominant TM mode in a waveguide filled with a material with $\varepsilon_r = 4$? The waveguide dimensions are $a = 2b = 5$ cm.

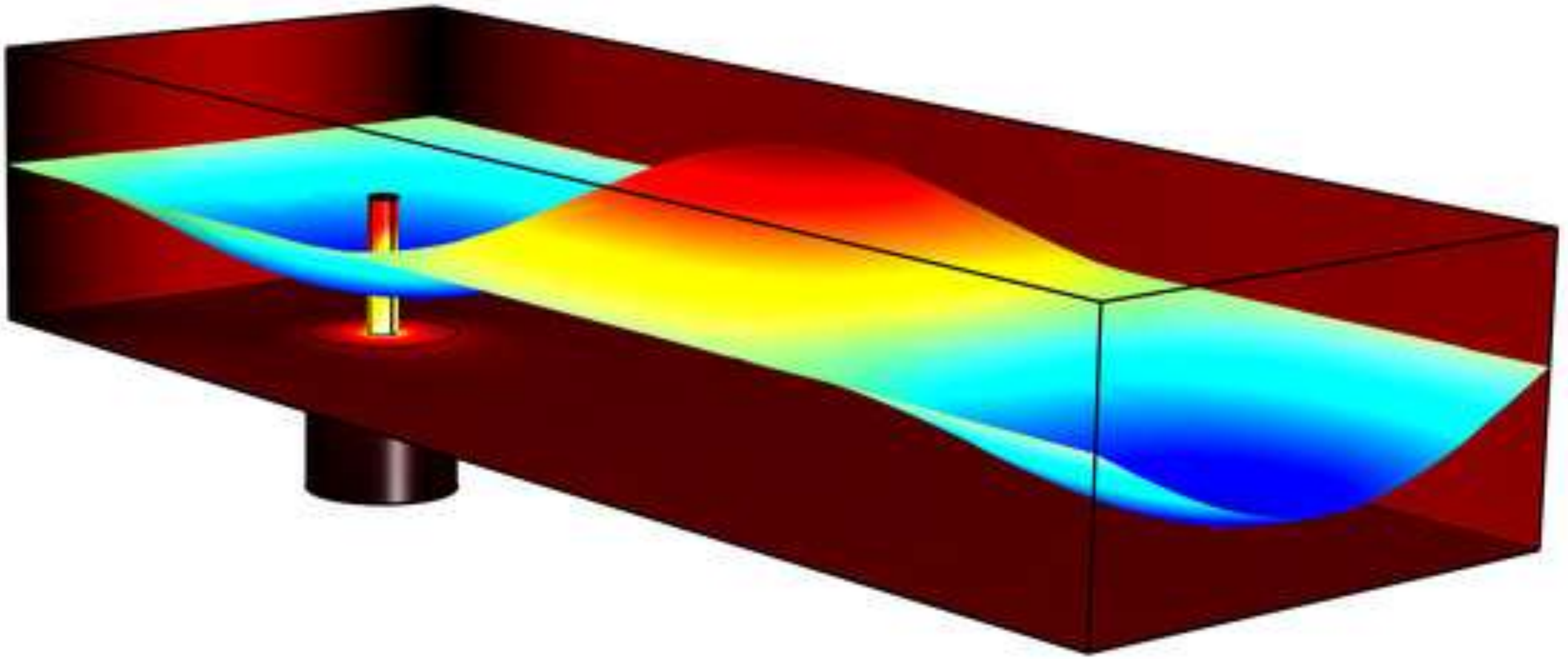
Solution: For TM_{11}, Eq. (8.106) gives

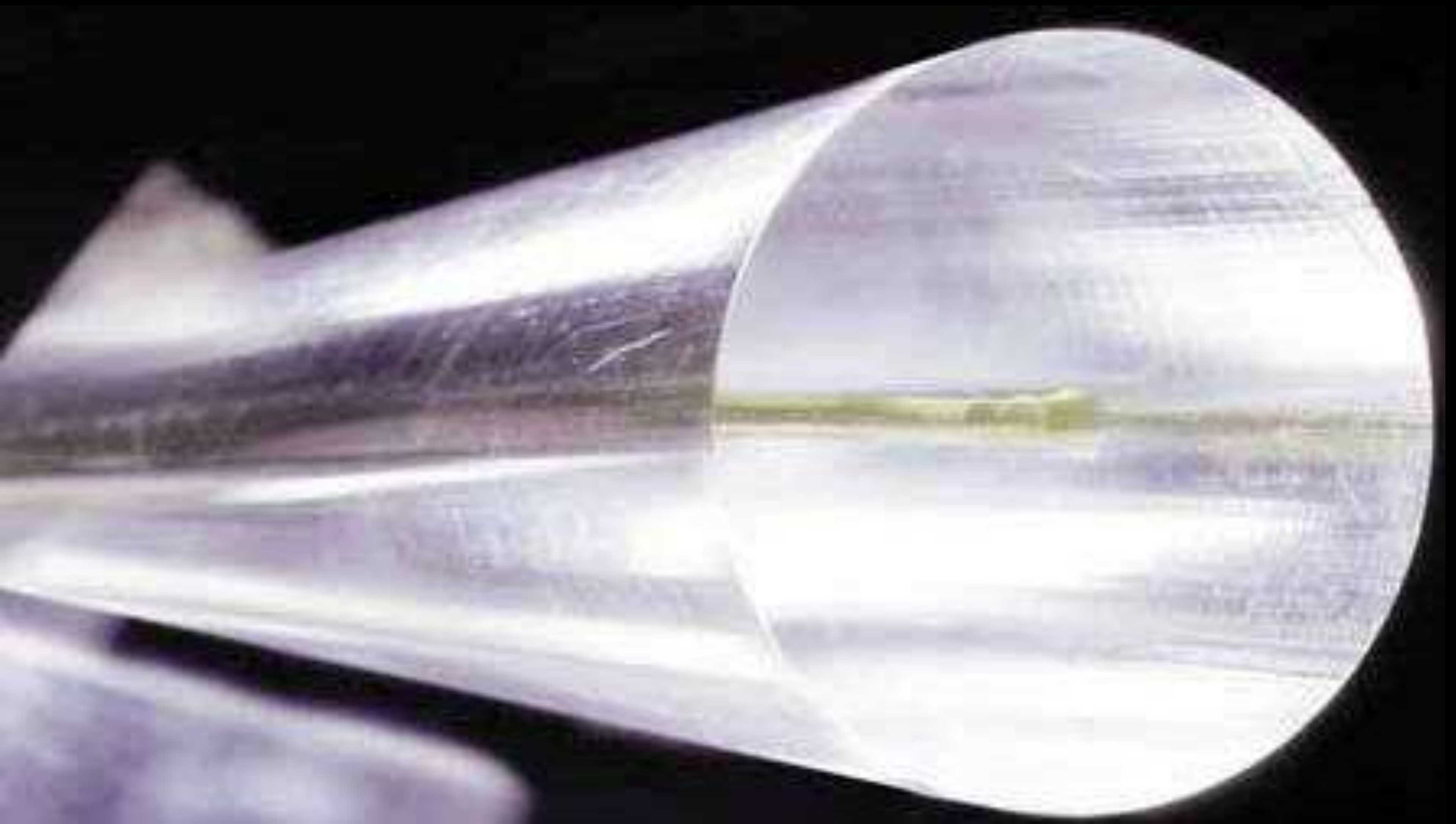
$$f_{11} = \frac{3 \times 10^8}{2\sqrt{4}} \left[\left(\frac{1}{5 \times 10^{-2}} \right)^2 + \left(\frac{1}{2.5 \times 10^{-2}} \right)^2 \right]^{1/2}$$

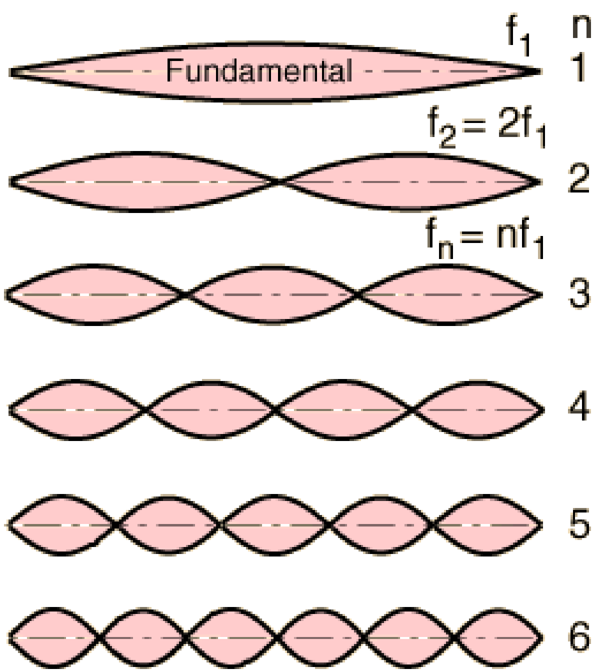
$$= 3.35 \times 10^9 \text{ Hz} = 3.35 \text{ GHz}.$$

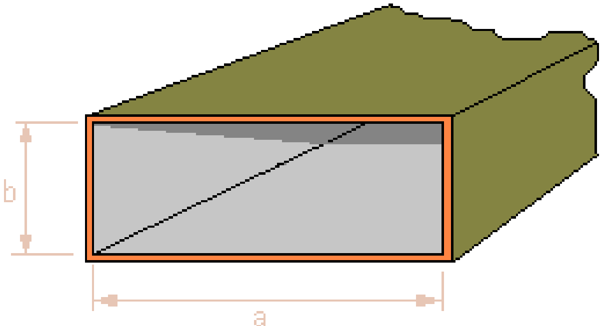
Wave properties for TE and TM modes in a rectangular waveguide with dimensions $a \times b$, filled with a dielectric material with constitutive parameters ε and μ. The TEM case, shown for reference, pertains to plane-wave propagation in an unbounded medium.

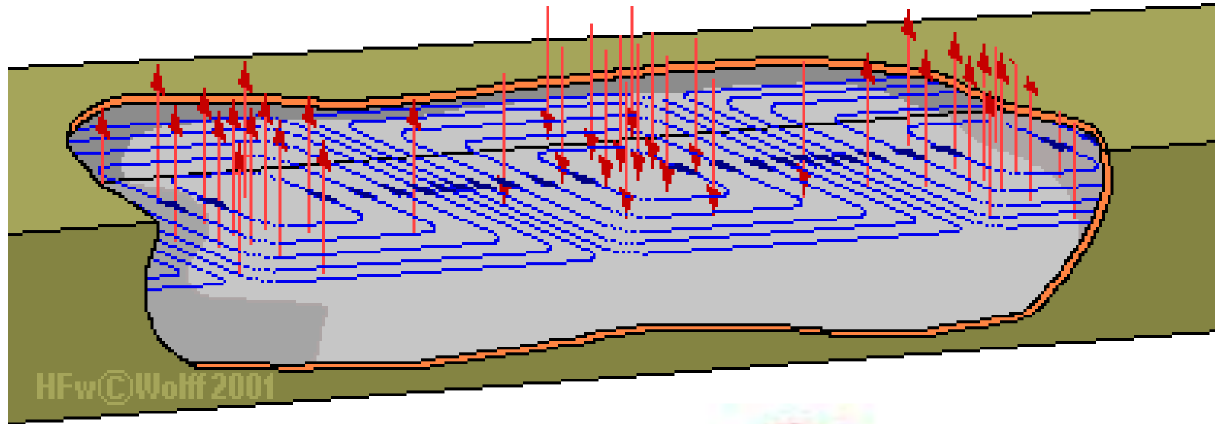
Rectangular Waveguides		Plane Wave
TE Modes	TM Modes	TEM Mode
$\widetilde{E}_x = \frac{j\omega\mu}{k_c^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{E}_x = \frac{-j\beta}{k_c^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{E}_x = E_{x0} e^{-j\beta z}$
$\widetilde{E}_y = \frac{-j\omega\mu}{k_c^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{E}_y = \frac{-j\beta}{k_c^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{E}_y = E_{y0} e^{-j\beta z}$
$\widetilde{E}_z = 0$	$\widetilde{E}_z = E_0 \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{E}_z = 0$
$\widetilde{H}_x = -Z_{\mathrm{TE}}\widetilde{E}_y$	$\widetilde{H}_x = -Z_{\mathrm{TM}}\widetilde{E}_y$	$\widetilde{H}_x = -\eta\widetilde{E}_y$
$\widetilde{H}_y = Z_{\mathrm{TE}}\widetilde{E}_x$	$\widetilde{H}_y = Z_{\mathrm{TM}}\widetilde{E}_x$	$\widetilde{H}_y = \eta\widetilde{E}_x$
$\widetilde{H}_z = H_0 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\, e^{-j\beta z}$	$\widetilde{H}_z = 0$	$\widetilde{H}_z = 0$
$Z_{\mathrm{TE}} = \eta \big/ \sqrt{1-(f_c/f)^2}$	$Z_{\mathrm{TM}} = \eta\sqrt{1-(f_c/f)^2}$	$\eta = \sqrt{\mu/\varepsilon}$
Properties Common to TE and TM Modes		
$f_c = \frac{u_{p0}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$		$f_c =$ not applicable
$\beta = k\sqrt{1-(f_c/f)^2}$		$k = \omega\sqrt{\mu\varepsilon}$
$u_p = \frac{\omega}{\beta} = u_{p0} \big/ \sqrt{1-(f_c/f)^2}$		$u_{p0} = 1/\sqrt{\mu\varepsilon}$



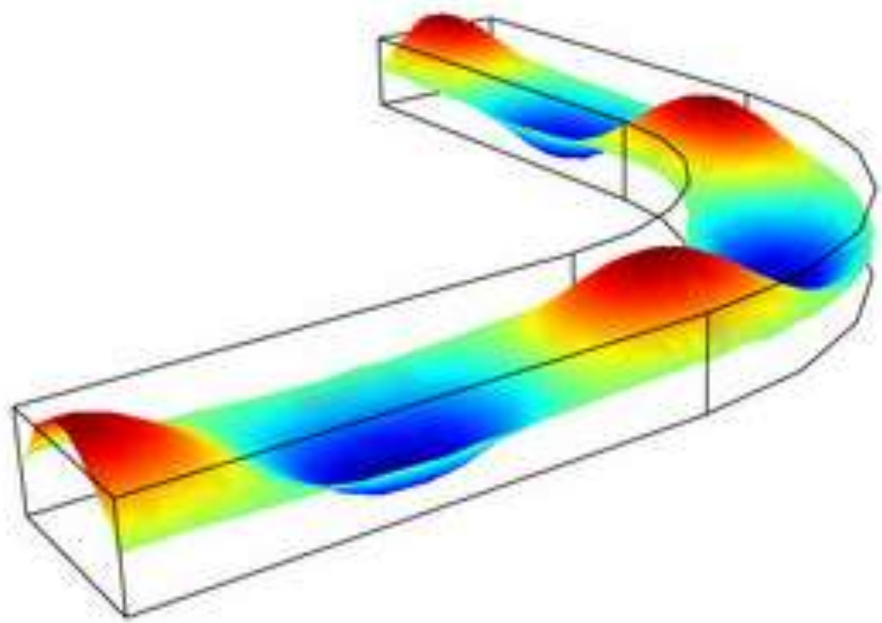



f1
n
Fundamental
1
f2 = 2f1
2
fn = nf1
3
4
5
6





$\text{Te}_{m,n}$ and $\text{TM}_{m,n}$



Properties Common to TE and TM Modes

$$f_c = \frac{u_{Po}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\beta = k\sqrt{1-(f_c/f)^2}$$

$$u_P = \frac{\omega}{\beta} = u_{Po}/\sqrt{1-(f_c/f)^2}$$

http://www.rfcafe.com/references/electrical/waveguide.htm

$U_p > U_{p_o} = C$ for air-filled wave guides

???

$U_p > U_{p_o} = C$ for air-filled wave guides

???

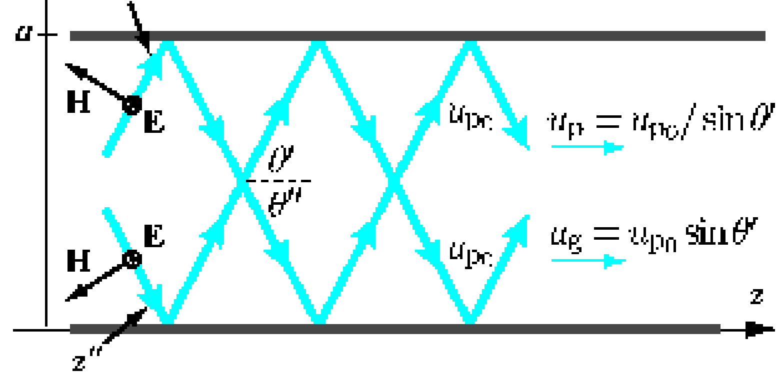
$$f_c = \frac{u_{P0}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

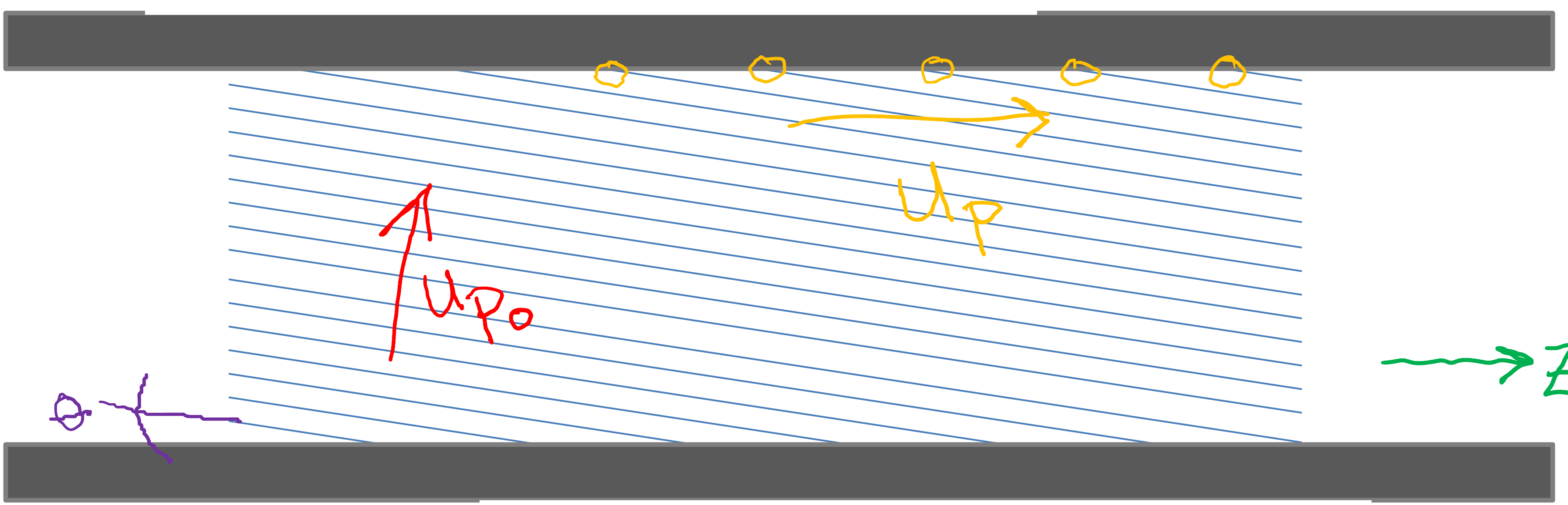
$$\beta = k\sqrt{1-(f_c/f)^2}$$

$$u_p = \frac{\omega}{\beta} = u_{P0}/\sqrt{1-(f_c/f)^2}$$

Phase speed down the waveguide ...

The TE_{10} mode can be constructed as the sum of two TEM waves.





A true UPW already exists at all place and time, so its *phase* speed doesn't violate Einstein.

A true *signal* is typically composed of infinitely many UPWs (EE 350 Rocks!!! ☺)
… it's the propagation of this <u>group</u> that carries info; the **group** velocity can't exceed c !!!

Phase Velocity

$$u_p = \frac{\omega}{\beta}$$

Group Velocity

$$u_g = \frac{1}{\partial\beta / \partial\omega}$$

(why not $\partial\omega / \partial\beta$??)

Note: this applies to <u>all</u> wave phenom…

Properties Common to TE and TM Modes

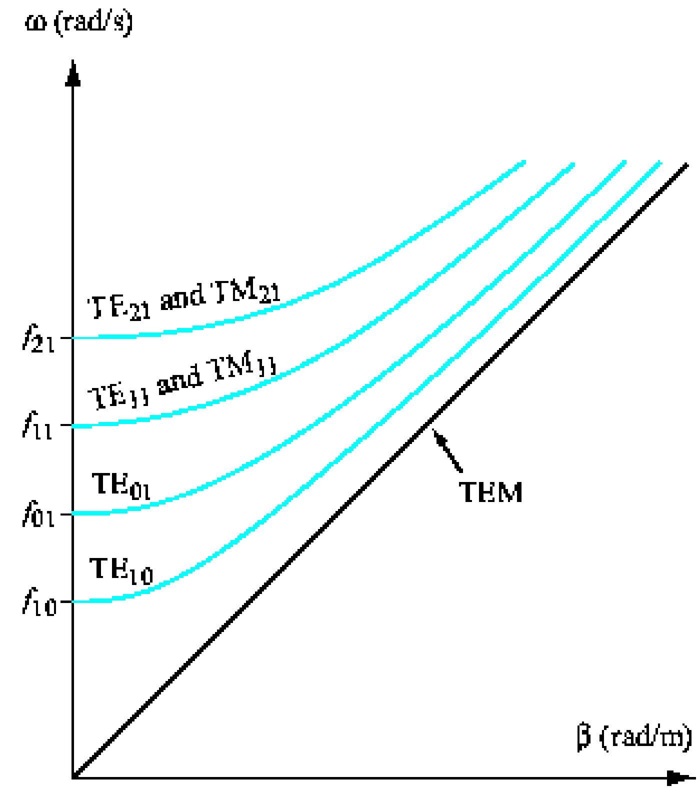
$$f_c = \frac{u_{p0}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\beta = k\sqrt{1-(f_c/f)^2}$$

$$u_p = \frac{\omega}{\beta} = u_{p0}/\sqrt{1-(f_c/f)^2}$$

$$u_g = u_{p0}\sqrt{1-(f_{mn}/f)^2}$$

$$u_p u_g = u_{p0}^2$$



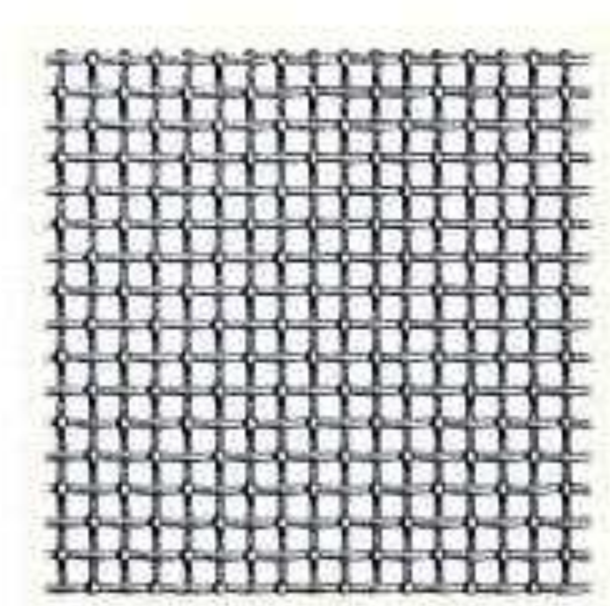
ω-β diagram for TE and TM modes in a hollow rectangular waveguide. The straight line pertains to propagation in an unbounded medium or on a TEM transmission line.

Exercise 8.11 What is the cutoff frequency for the dominant TM mode in a waveguide filled with a material with $\varepsilon_r = 4$? The waveguide dimensions are $a = 2b = 5$ cm.

Solution: For TM_{11}, Eq. (8.106) gives

$$f_{11} = \frac{3 \times 10^8}{2\sqrt{4}} \left[\left(\frac{1}{5 \times 10^{-2}} \right)^2 + \left(\frac{1}{2.5 \times 10^{-2}} \right)^2 \right]^{1/2}$$

$$= 3.35 \times 10^9 \text{ Hz} = 3.35 \text{ GHz}.$$



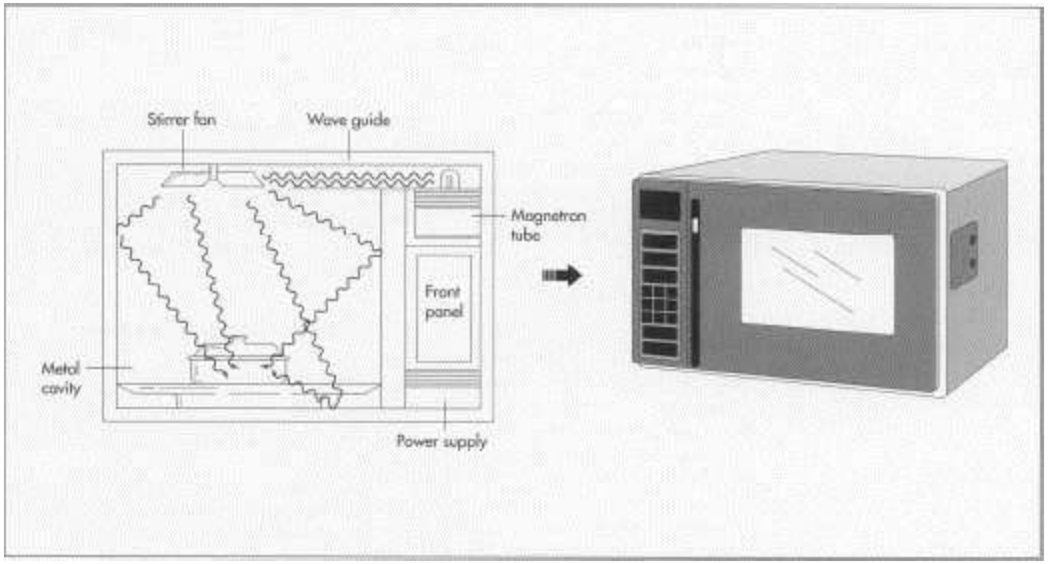
Tacoma Narrows Bridge

http://www.youtube.com/watch?v=P0Fi1VcbpAI

Resonance

Wine Glass

http://www.youtube.com/watch?v=17tqXgvCN0E&feature=related



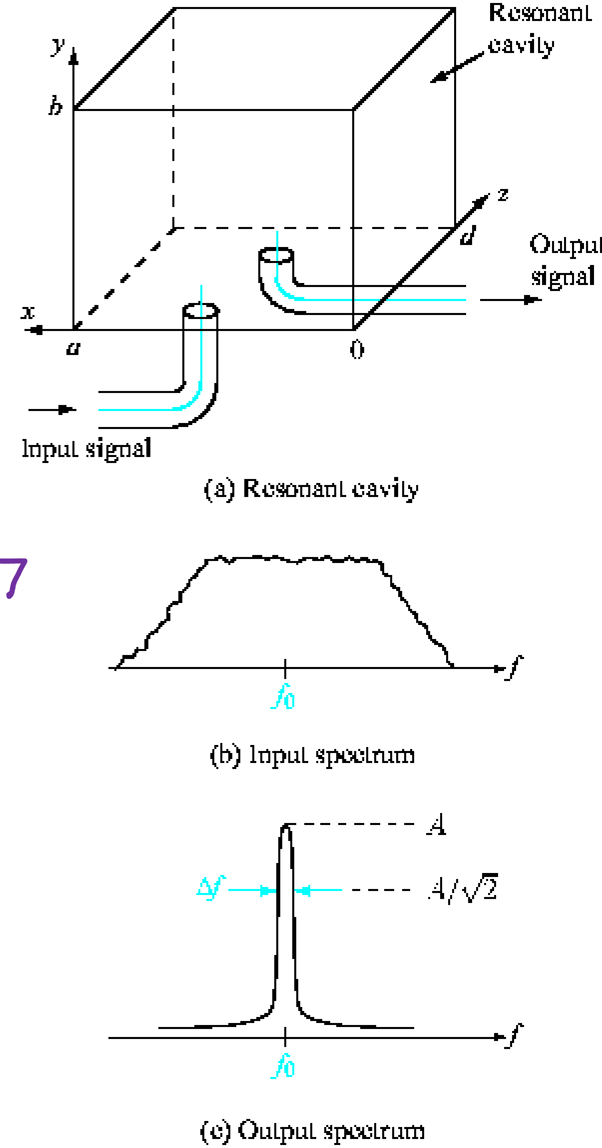


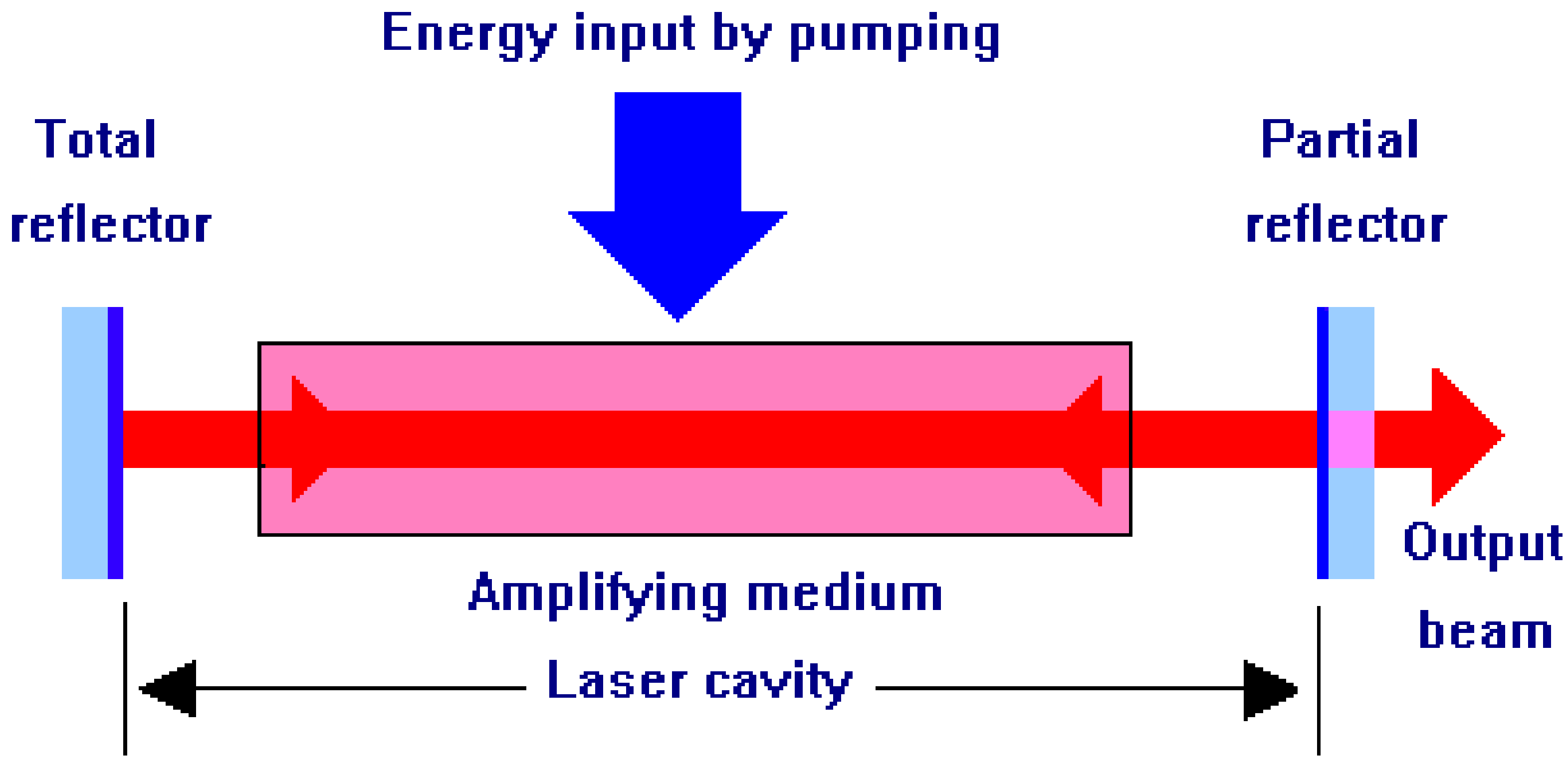
Figure 8-28: A resonant cavity supports a very narrow bandwidth around its resonant frequency f_0.

$$f_{mnp} = \frac{u_{p0}}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$$

Length of cavity

with

Quality FACTOR $\quad Q \approx \dfrac{f_{mnp}}{\Delta f}$


Energy input by pumping
Total
reflector
Partial
reflector
Output
beam
Amplifying medium
Laser cavity